

Title: Go With The Flow!**Brief Overview:**

In this three-lesson unit, students will be introduced to flow chart proofs. Students will become familiar with the flow chart format by using them to organize real life scenarios, and then use what they have learned to prove triangle congruence. The methods within the lessons stress logically sequencing definitions, postulates and theorems, as well as finding the essential elements necessary to lead into one of the triangle congruence postulates. It is assumed that students have experience with SSS, SAS, AAS, ASA and HL postulates.

NCTM Content Standard/National Science Education Standard:

Geometry

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grade/Level:

Grades 9 – 12, Geometry

Duration/Length:

Three 90 minute periods.

Student Outcomes:

Students will be able to:

- Use logical reasoning to complete triangle congruency proofs.
- Analyze triangles in order to identify essential information and postulate to complete a proof.

Materials and Resources:

- Flow chart and worksheets on overhead transparency sheets
- Overhead markers
- Sentence strips with magnets or tape
- Scissors
- Glue sticks or tape

- Worksheets:
 - Real Life Proofs
 - Identifying Triangle Congruency
 - Flow Chart Proofs #1 – 5
 - Cut ‘n Paste template
 - Missing Parts
 - Flow Chart Proofs #6 – 14
 - Two Column Proofs
 - Proofs #15 – 19
 - More Two Column Proofs
 - Flow Chart Template
 - Round Table Proofs

Development/Procedures:

Lesson 1

Launch – Introduce the idea of flow chart proof using the “making a peanut butter and jelly sandwich” example from the “Real Life Proofs” worksheets.

To use the flow chart, “preliminary information” goes in the dotted ellipses. This information does not contribute directly to forming the sandwich. “Essential steps/ingredients” go in the three solid squares below the dotted ellipses. Reasons for each step go on the lines below each shape.

In this example, students do not need to use the fourth dotted ellipse, for the proof is complete after the step where students eat the sandwich.

Continue with the “writing a research paper” from “Real Life Proofs”, allowing students to volunteer the majority of inputs. Allow students to complete “going to the mall” in pairs at their seats. Note that answers for these proofs don’t need to match the provided answer key exactly, but students should have the general idea presented in the solutions. After reviewing their solutions, students should pose their own givens and ultimate goals to their partners and challenge them to complete a flow chart proof.

Preassessment – Distribute “Identifying Triangle Congruency” for postulate and theorem review.

Teacher Facilitation – Model completing a triangle congruence proof with given statements and reasons, using “Flow Chart Proof #1”. The instructor should link these proofs with the previous real life examples by emphasizing that “essential ingredients” to a triangle congruence proof are the angles and sides that are

congruent (e.g. SAS). This information is always placed in the solid squares. Any “preliminary information” will be information which leads to essential information, and is written in the dotted ellipses. For example, “preliminary information” might be that an angle bisector exists, which will lead to the “essential information” of two angles being congruent. The fourth solid square will state the triangles now proven congruent by the essential information. The blank underneath the fourth square will be filled in with the appropriate triangle congruency theorem or postulate.

For “Flow Chart Proof #2”, give students statements and reasons to cut and paste from the “Cut ‘n Paste” template. Use magnetic sentence strips to model how to match the “Cut ‘n Paste” strips into the flow chart as students complete the proof at their desks. Encourage students to first sort through the strips to identify those that are reasons, and those that are statements. It is also helpful to identify which strips are the given information, and which strips are added information. For an added visual technique, project a flow chart on the board and put sentence strips on board within the projected image of the flow chart. Stress to students that the dotted circles on the flow chart may or may not need to be used. These are for preliminary information that does not contribute directly to a triangle congruence theorem, but can be used as stepping stones to the facts that one can use to prove triangles congruent.

Student Application –Distribute the remaining proofs from “Flow Chart Proofs #1 – 5”. Student should cut out sentence strips for each proof, sort them, and then complete the proofs. They should only do one proof at a time to avoid mixing up statements and reasons from each proof. Take appropriate breaks to review solutions to the proofs on the board, and determine breaks based upon the understanding of the students.

Embedded Assessment – Partway through class, the instructor can ask groups of students to put their solutions to proofs on the board or present them to the class. Classmates can ask each other questions based on their presentations to clarify any confusion.

Reteaching/Extension –

- Encourage students who finish the four cut and paste proofs to redo proof #5 without cutting and pasting, trying to find an alternative postulate to prove congruency. Refer to the answer key for Proof #5 for alternative solutions. They will be asked to find a second solution to the problem.

- For those students who have not finished their original proofs, they may continue to work on them with increased instructor assistance

Lesson 2

Preassessment – Warm up with the worksheet “Missing Parts”, which asks students to identify missing information necessary to prove triangles congruent.

Launch – Students will complete Proof #6 in pairs, since they will have to complete the proof without cut-out strips.

Teacher Facilitation – Use proof # 6 to introduce into corresponding parts of congruent triangles are congruent (CPCTC). Ask students why they can make the statement that $\overline{JM} \cong \overline{KM}$. Students should respond or be led to the reason CPCTC. Inform students that this is entered in the last dotted ellipse, and stress the importance of the *flow*. That is, CPCTC cannot be assumed until congruency has been proven.

Student Application – Students will complete Proofs #7-14 using flow charts.

Embedded Assessment – Each group will present one of the proofs on overhead.

Reteaching/Extension –

- Students experiencing difficulty can finish their proofs with increased instructor assistance.
- Students who have finished their proofs with a good understanding will expand on their ideas in their presented proof to create their own form of proof (e.g. a proof using something other than a flow chart).
- Students should discuss their ideas in small groups; the instructor should not lead them to any conclusions. The instructor should simply ask if students can rewrite the proof without the boxes, circles and arrows, such that a third party could read and understand the statements and reasons in order.

Lesson 3

Preassessment – Distribute “Two Column Proofs”, in which students will complete two proofs in two-column form, one of which has all the statements and the reasons are missing and the other which supplies the reasons and asks for the statements.

Launch – Have the students present and discuss their solutions, and compare and contrast the two-column form to the flow chart. This

discussion can be in the form of a “think-pair-share” focused around the questions “Which form of proof do you prefer and why?” and “Which form of proof do you think a third party would find easiest to understand and why?”

Teacher Facilitation – Discuss the alternative forms of proof completed during “reteaching/ extension”. Students may or may not have suggested two-column proofs. If so, allow the student to explain their reasoning and methods. The students may also have suggested additional valid proof methods; encourage them to share their ideas with the class.

Student Application – Students will complete Proofs #15-19 using the proof method of their choice. Encourage students to use more than one method on all proofs. For those that choose flow charts, provide copies of the “Flow Chart Template.”

Embedded Assessment – Organize students into groups of 3 or 4 in a circle, or a shape that allows passing of paper. Each individual is given a half a sheet of paper with a two-column proof from “Round Table Proofs” worksheet. Each proof has six pieces of information missing. Each student should write their name at the top of the paper, and draw a picture describing the given information. Then they pass their paper to the person to their left who writes their name at the top and completes the first line of missing information. They pass it to the next member, and this continues until the proof is completed. After the last line is completed, the paper should be returned to its original owner, who will be responsible for checking the group’s answers. If they wish to make changes they must discuss changes as a group.

Reteaching/Extension –

- Students should write 3 – 4 sentences about how they work through proofs. Prompt students, asking them, “What’s the first thing you do when you see a proof?”, “How do you know what comes first, or last?”, and “Which proof format do you now think is the best, and why?”

Summative Assessment:

The instructor should instruct students to combine proofs #1-19 for a portfolio on proofs. In addition they should create their own proof, providing a labeled drawing of two triangles, givens, statements and reasons mixed up and an answer key.

Authors:

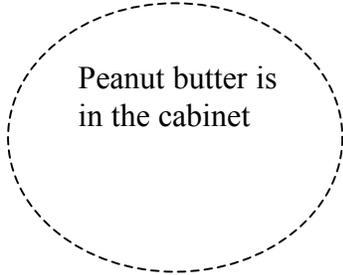
Madeline Ahearn
Baltimore Freedom Academy
Baltimore, MD

Marla Sanders
New Town High School
Owings Mills, MD

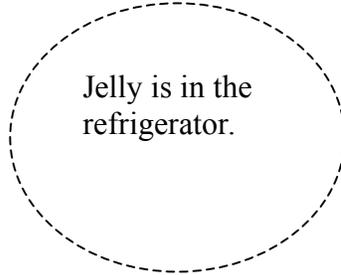
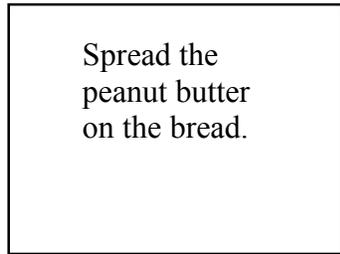
Real Life Proofs

Given: You have peanut butter in your cabinet, jelly in your refrigerator.

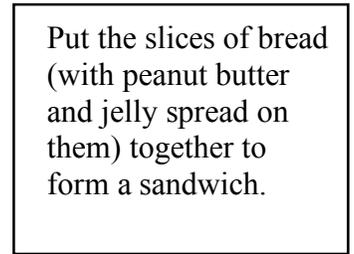
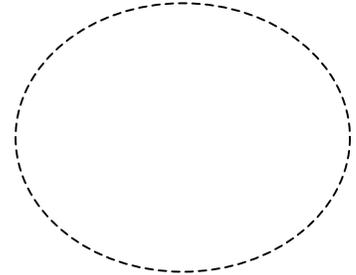
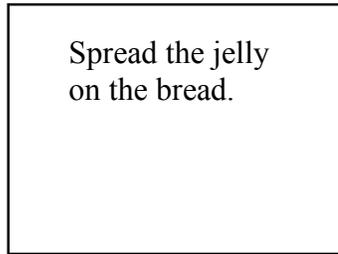
Ultimate Goal: You want to eat a peanut butter and jelly sandwich.



Given



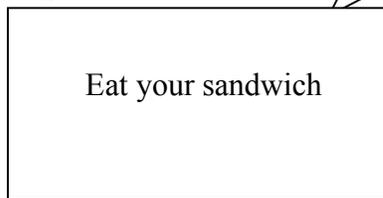
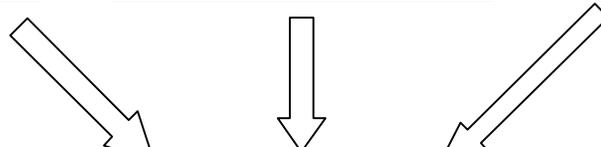
Given



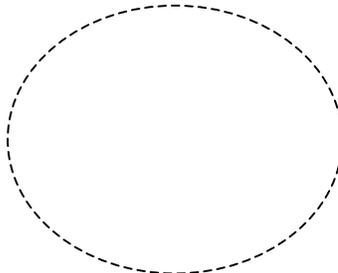
PB is necessary for a PBJ sandwich

Jelly is needed for a PBJ sandwich

Definition of sandwich

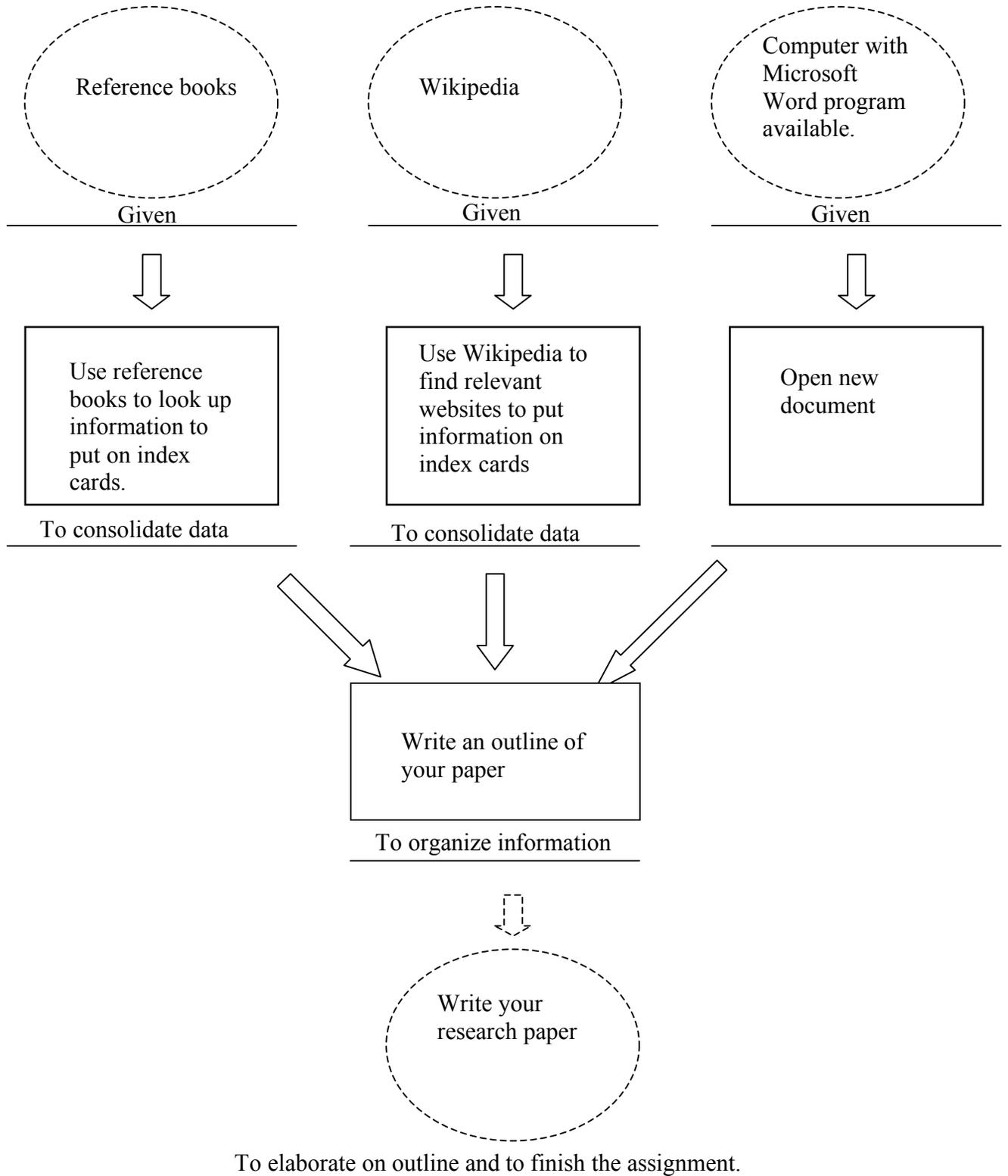


You're hungry

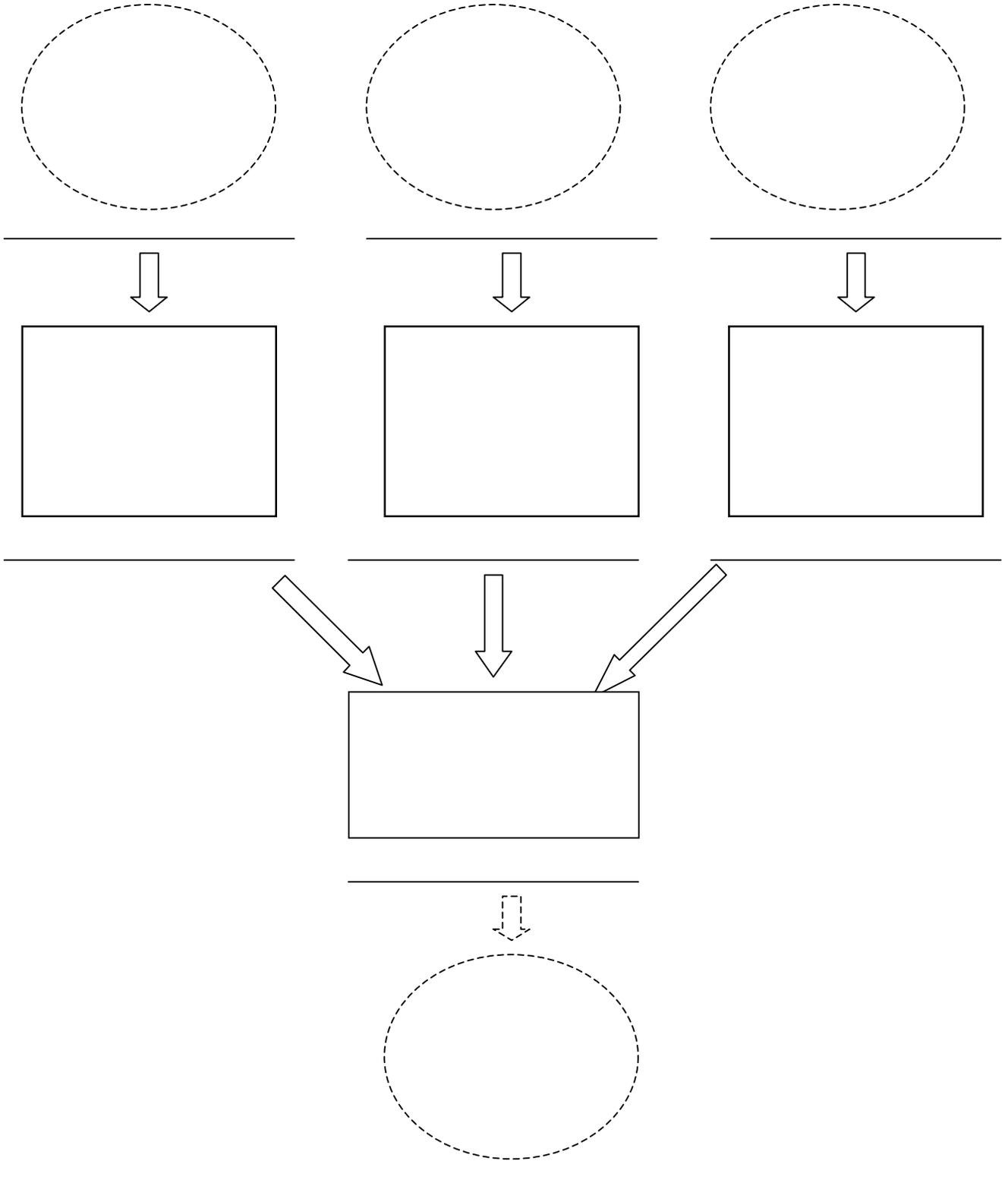


Given: You have reference books, Wikipedia, and a computer with Microsoft Word software.

Ultimate Goal: You need to write a research paper.

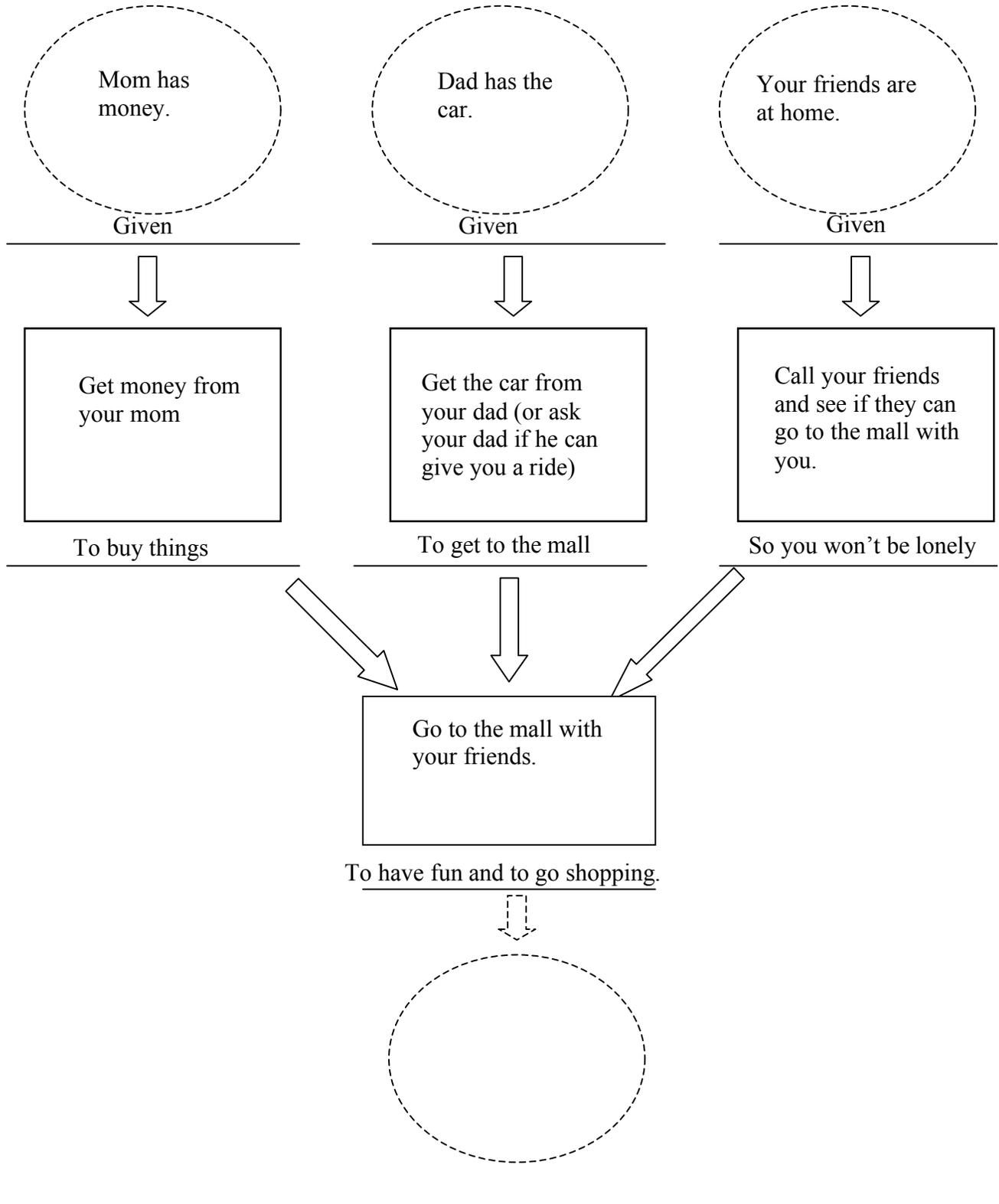


Given: Your mom has money, your dad has a car, and your friends are at home
Ultimate Goal: You want to go to the mall



Given: Your mom has money, your dad has a car, and your friends are at home
Ultimate Goal: You want to go to the mall

ANSWER KEY



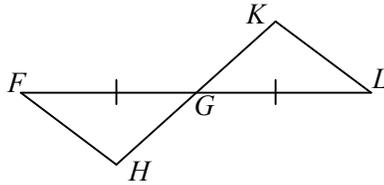
Identifying Triangle Congruency

Name: _____

Date: _____

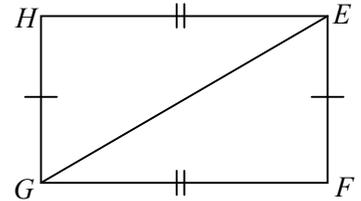
For each example, choose the proper triangle congruency postulate theorem to use to prove triangles congruent.

1.

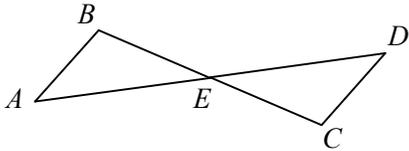


$$\overline{FH} \parallel \overline{KL}$$

2.



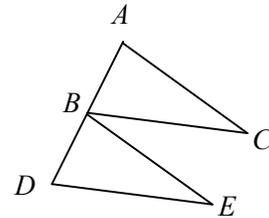
3.



$$\overline{AE} \cong \overline{DE}$$

$$\overline{BE} \cong \overline{CE}$$

4.

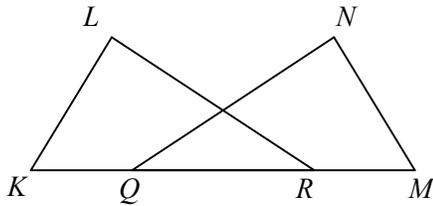


B is the midpoint of \overline{AD}

$$\overline{BC} \parallel \overline{DE}$$

$$\angle C \cong \angle E$$

5.

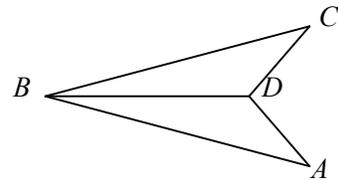


$$\angle MQN \cong \angle KRL$$

$$\overline{KQ} \cong \overline{MR}$$

$$\angle K \cong \angle M$$

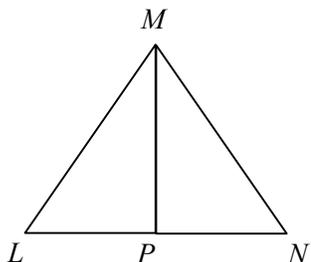
6.



$$\angle ABD \cong \angle CBD$$

$$\angle A \cong \angle C$$

7.



$\triangle LMP$ and $\triangle NMP$ are right triangles

P is the midpoint of \overline{LN}

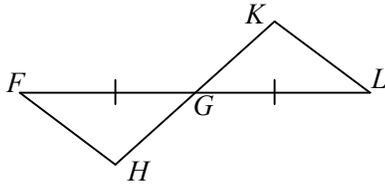
Identifying Triangle Congruency

Name: ANSWER KEY

Date: _____

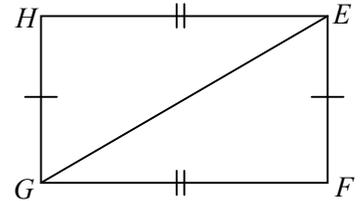
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1.



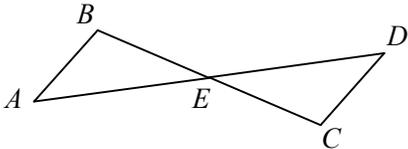
$\overline{FH} \parallel \overline{KL}$ ANSWER: ASA

2.



ANSWER: SSS

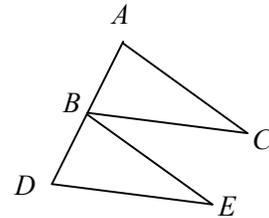
3.



$\overline{AE} \cong \overline{DE}$
 $\overline{BE} \cong \overline{CE}$

ANSWER: SAS

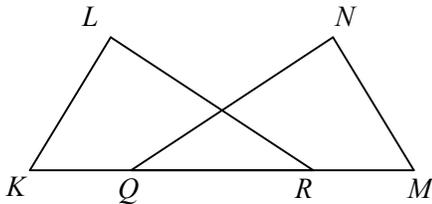
4.



B is the midpoint of \overline{AD}
 $\overline{BC} \parallel \overline{DE}$
 $\angle C \cong \angle E$

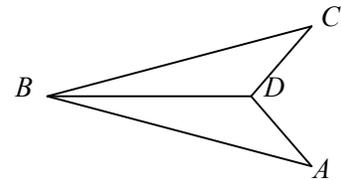
ANSWER: AAS

5.



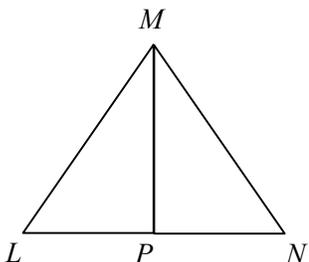
$\angle MQN \cong \angle KRL$
 $\overline{KQ} \cong \overline{MR}$
 $\angle K \cong \angle M$ ANSWER: ASA

6.



$\angle ABD \cong \angle CBD$
 $\angle A \cong \angle C$
 ANSWER: AAS

7.



$\triangle LMP$ and $\triangle NMP$ are right triangles
 P is the midpoint of \overline{LN}

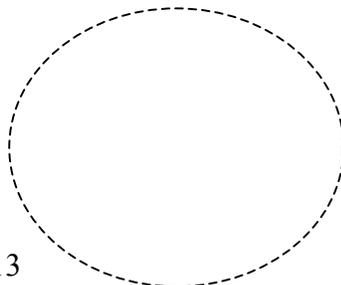
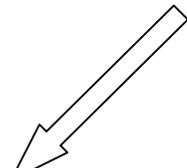
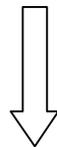
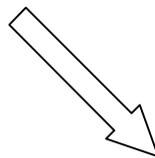
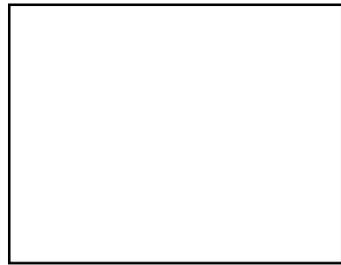
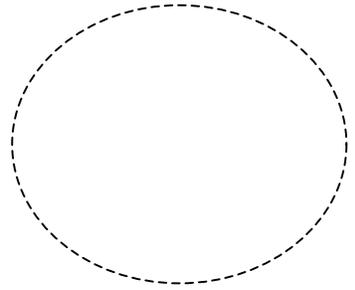
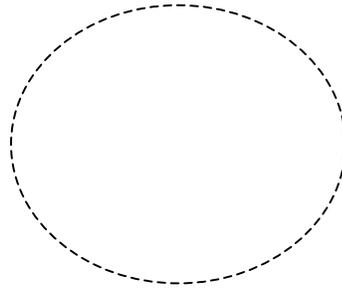
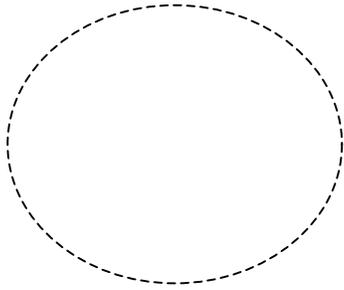
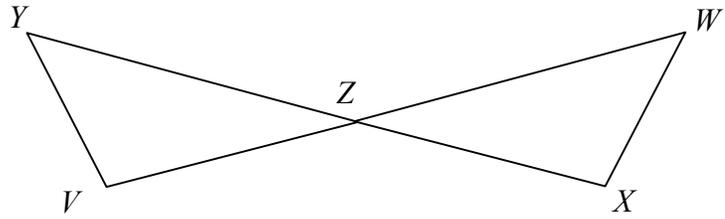
ANSWER: HL or SAS

Flow Chart Proofs #1 – 5

Proof #1

Given: $\overline{YZ} \cong \overline{WZ}$
 $\overline{ZV} \cong \overline{ZX}$

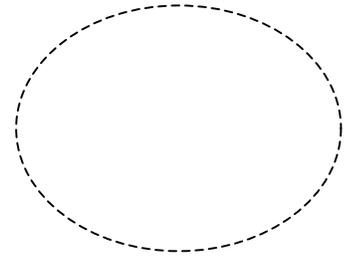
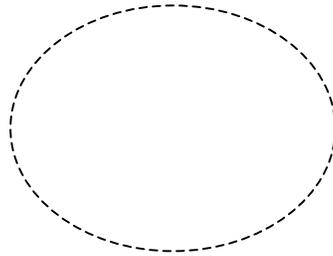
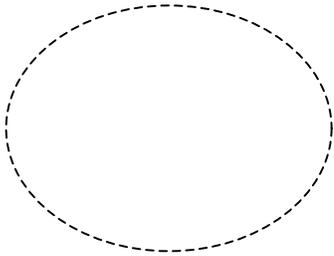
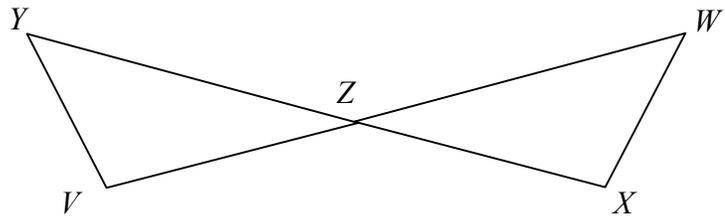
Prove: $\triangle VYZ \cong \triangle XWZ$



Flow Chart Proofs #1 – 5
Proof #1 ANSWER KEY

Given: $\overline{YZ} \cong \overline{WZ}$
 $\overline{ZV} \cong \overline{ZX}$

Prove: $\triangle VYZ \cong \triangle XWZ$



$\overline{YZ} \cong \overline{WZ}$

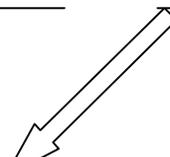
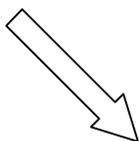
$\overline{ZV} \cong \overline{ZX}$

$\angle YZV \cong \angle WZX$

Given

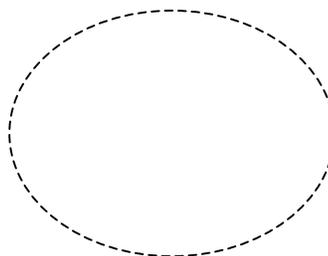
Given

Vertical Angles



$\triangle VYZ \cong \triangle XWZ$

SAS

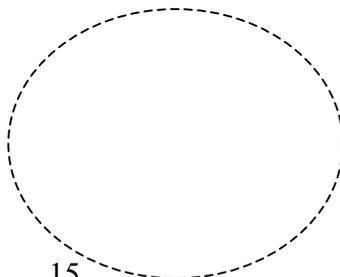
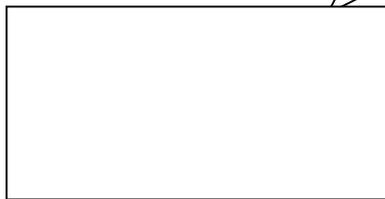
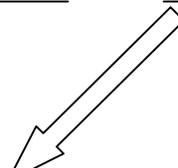
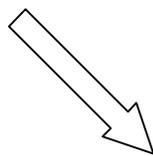
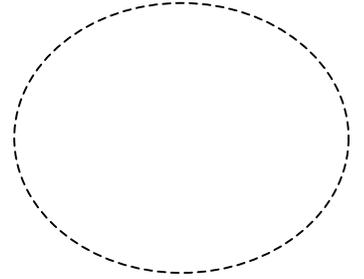
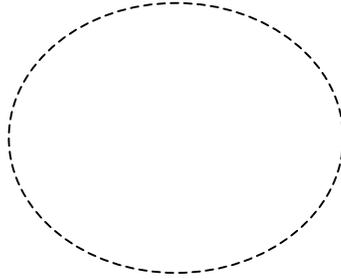
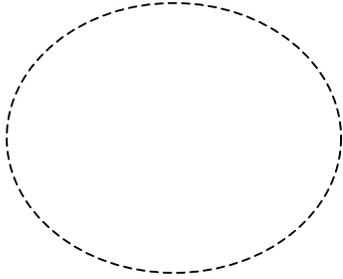
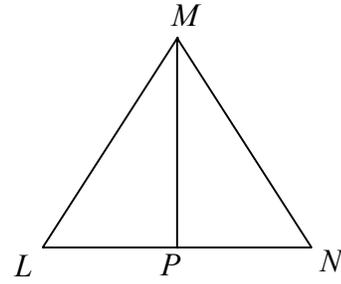


Proof #2

Given: $\angle MPL$ and $\angle MPN$ are right angles

$$\overline{LP} \cong \overline{NP}$$

Prove: $\triangle LMP \cong \triangle NMP$

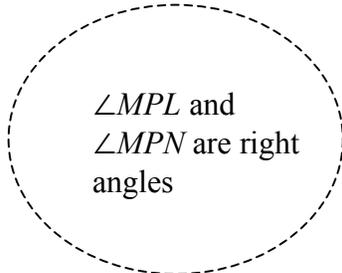
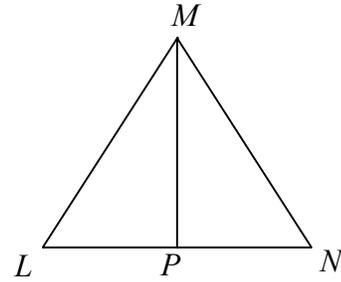


Proof #2 ANSWER KEY

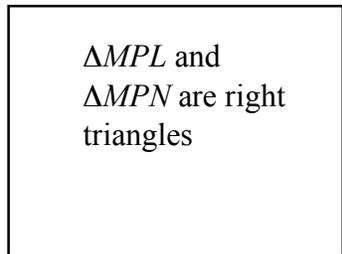
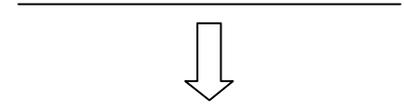
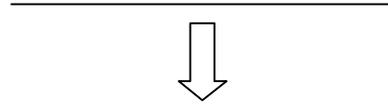
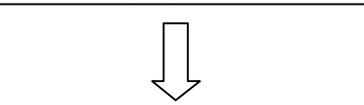
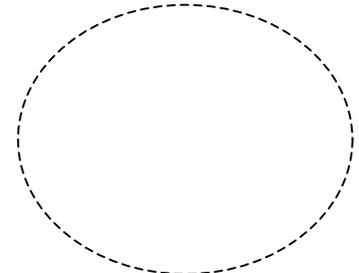
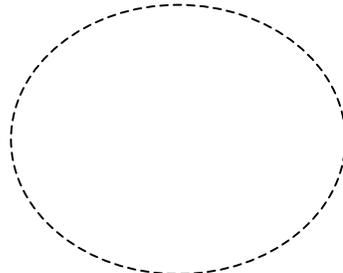
Given: $\angle MPL$ and $\angle MPN$ are right angles

$$\overline{LP} \cong \overline{NP}$$

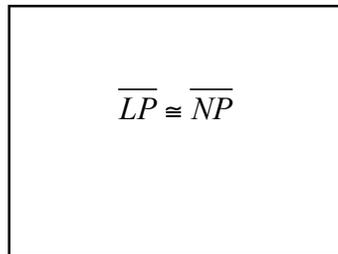
Prove: $\triangle LMP \cong \triangle NMP$



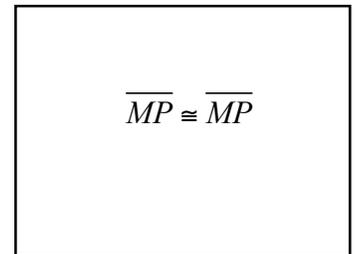
Given



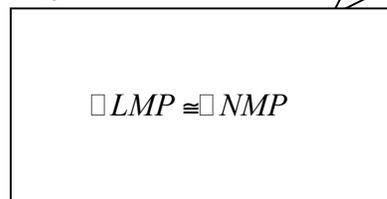
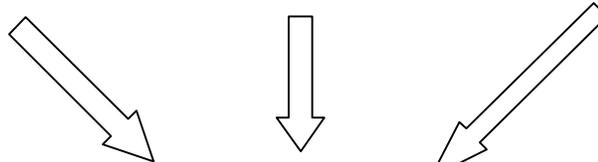
Defn. of a right triangle



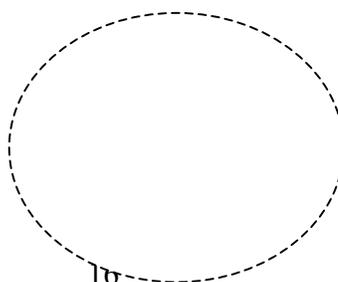
Given



Reflexive Prop

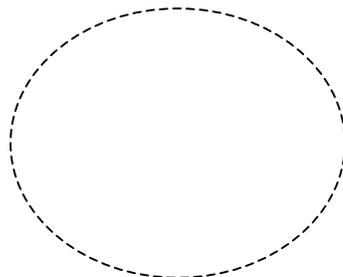
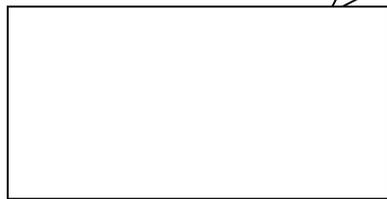
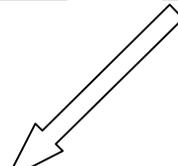
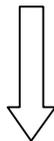
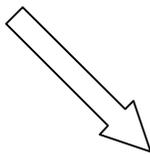
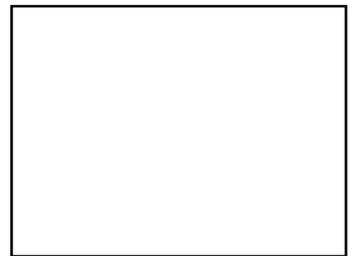
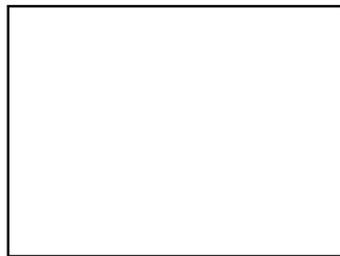
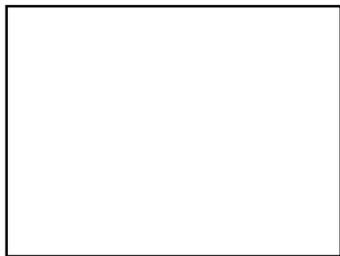
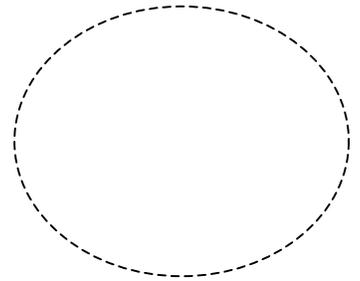
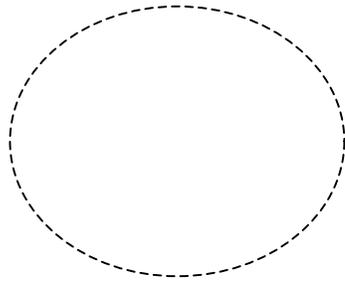
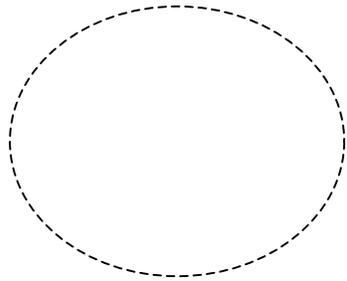
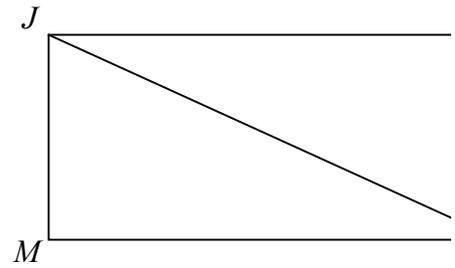


HL



Proof #3

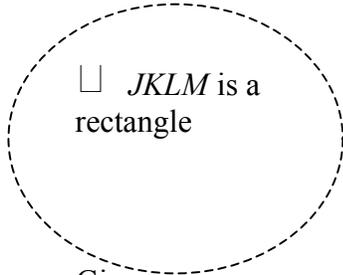
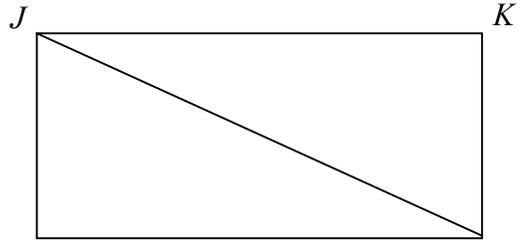
Given: $\square JKLM$ is a rectangle
Prove: $\triangle JLM \cong \triangle LJK$



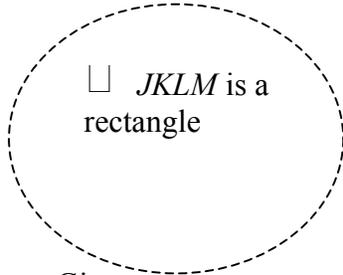
Proof #3 ANSWER KEY

Given: $\square JKLM$ is a rectangle

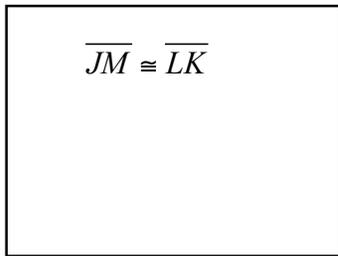
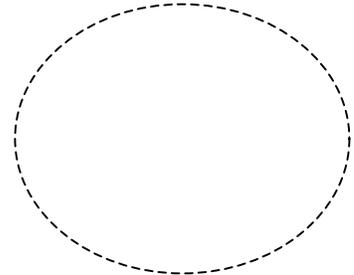
Prove: $\square JLM \cong \square LJK$



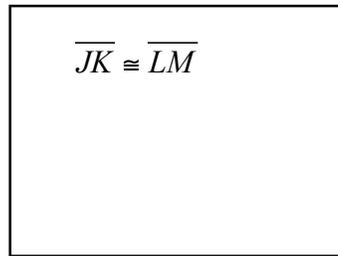
Given



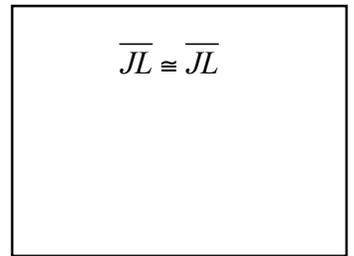
Given



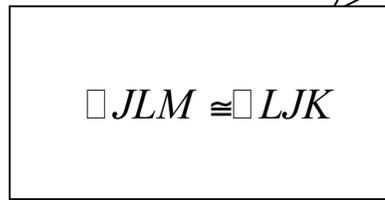
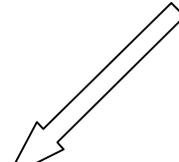
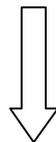
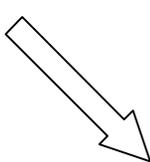
Opp. sides of a rect. are congruent



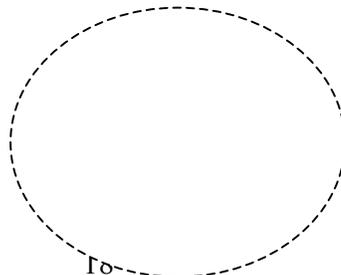
Opp. sides of a rect. are congruent



Reflexive Prop.



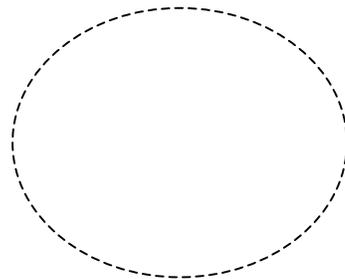
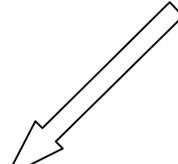
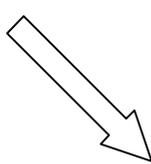
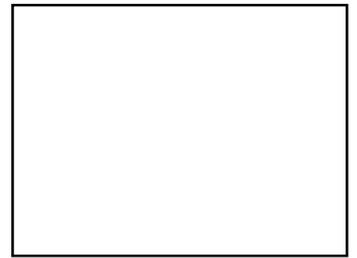
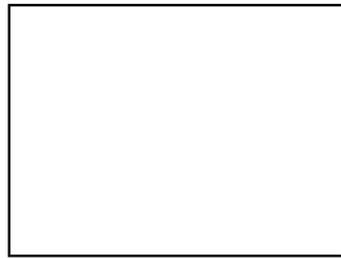
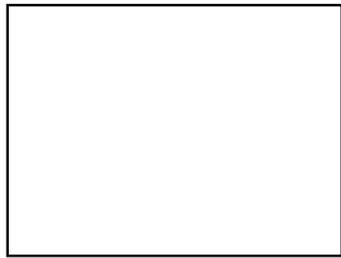
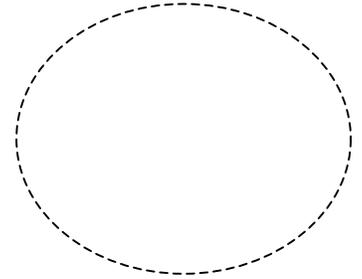
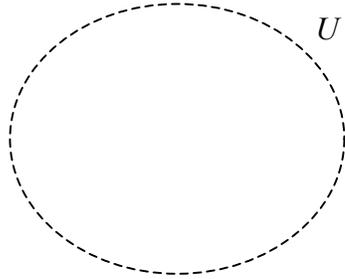
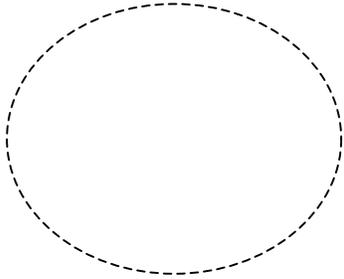
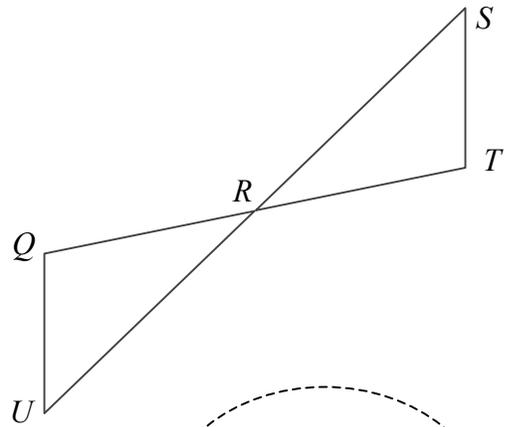
SSS



Proof #4

Given: $\overline{QU} \parallel \overline{TS}$, R is the midpoint of \overline{QT}

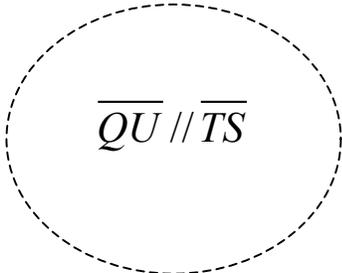
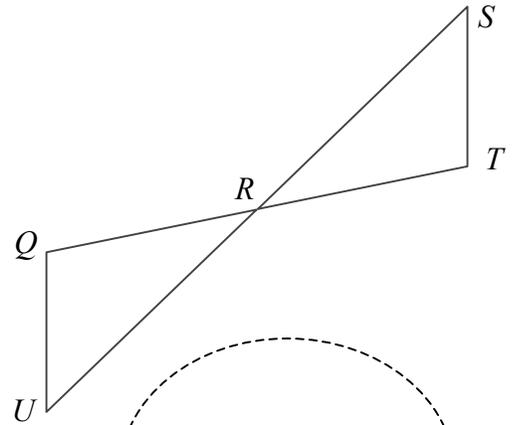
Prove: $\triangle RQU \cong \triangle RTS$



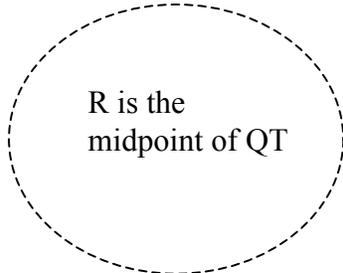
Proof #4 ANSWER KEY

Given: $\overline{QU} \parallel \overline{TS}$, R is the midpoint of \overline{QT}

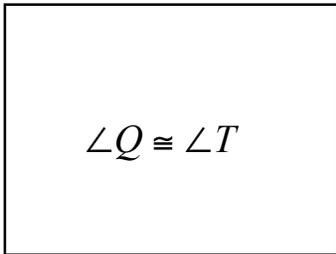
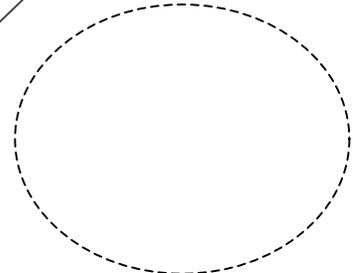
Prove: $\triangle RQU \cong \triangle RTS$



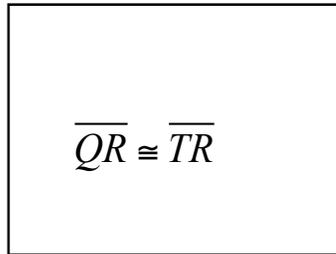
Given



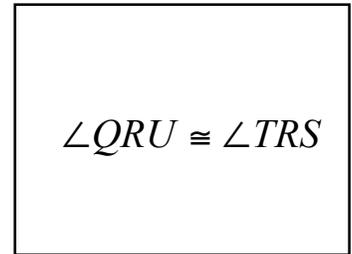
Given



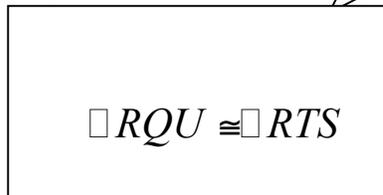
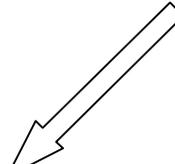
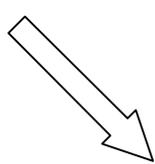
Alternate Interior Angles



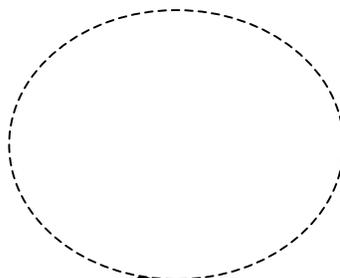
Definition of a Midpoint



Vertical Angles



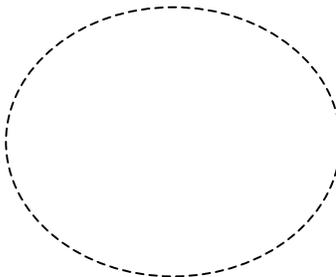
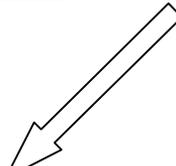
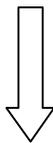
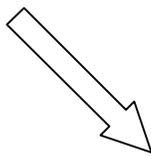
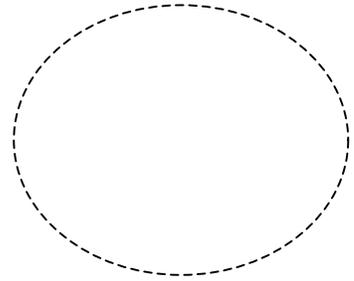
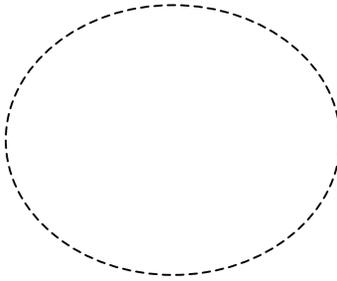
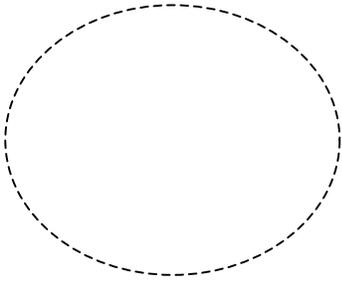
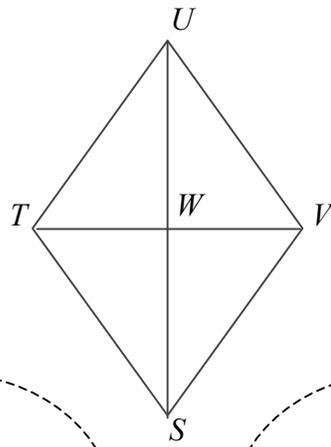
ASA



Proof #5

Given: $\square STUV$ is a parallelogram

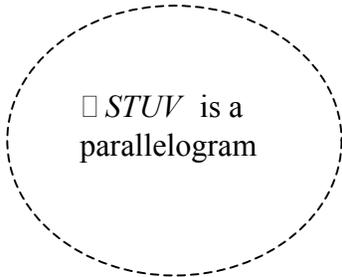
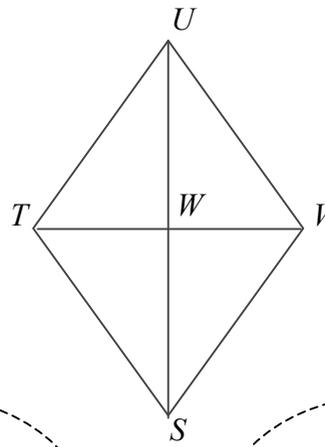
Prove: $\triangle UTW \cong \triangle SVW$



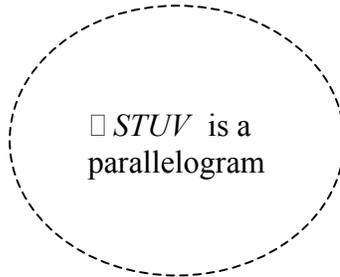
Proof #5 ANSWER KEY

Given: $\square STUV$ is a parallelogram

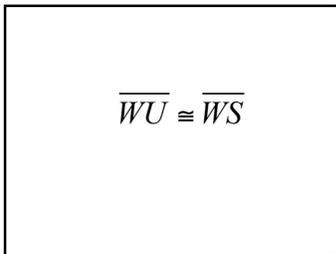
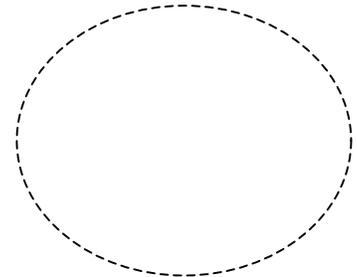
Prove: $\square UTW \cong \square SVW$



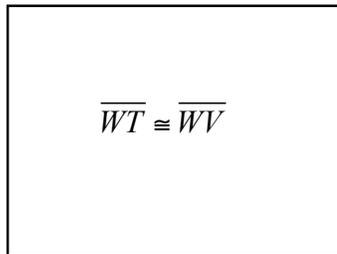
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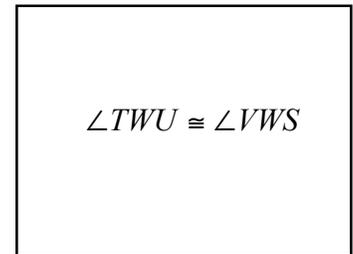
Given



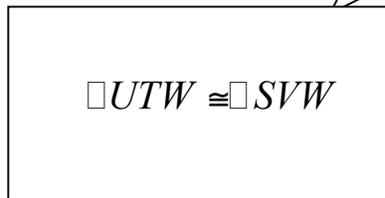
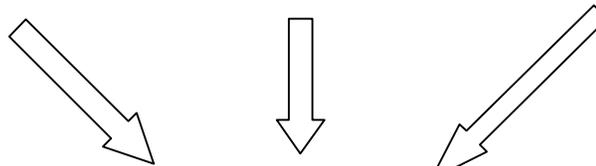
Diags of a //ogram bisect each other



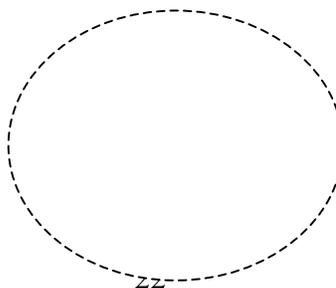
Diags of a //ogram bisect each other



Vertical Angles



SAS



Cut 'n Paste Template

Cut N' Paste – Proof #2

$\square LMP \cong \square NMP$	$\overline{MP} \cong \overline{MP}$	Given
$\angle MPL$ and $\angle MPN$ are right angles	Given	Defn. of a right triangle
HL	$\overline{LP} \cong \overline{NP}$	Reflexive Prop.
$\triangle MPL$ and $\triangle MPN$ are right triangles		

Cut N' Paste = Proof #3

$\overline{JM} \cong \overline{LK}$	$\square JLM \cong \square LJK$	Given
Opp. sides of a rect. are congruent	$\overline{JK} \cong \overline{LM}$	\square JKLM is a rectangle
SSS	Opp. sides of a rect. are congruent	Reflexive Prop.
$\overline{JL} \cong \overline{JL}$	\square JKLM is a rectangle	Given

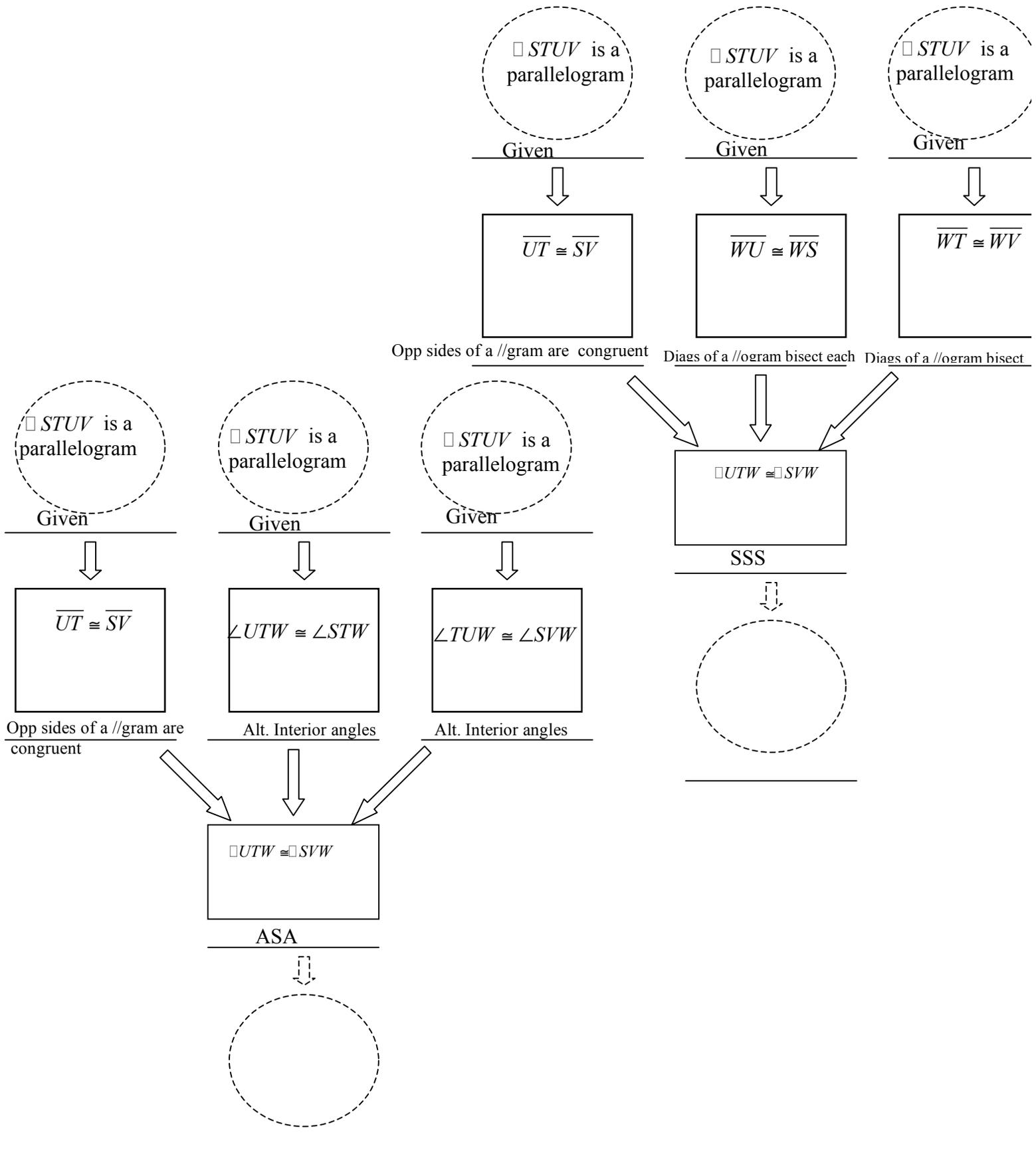
Cut N' Paste = Proof #4

$\square RQU \cong \square RTS$	$\angle QRU \cong \angle TRS$	Given
Definition of a Midpoint	ASA	$\overline{QR} \cong \overline{TR}$
$\angle Q \cong \angle T$	Vertical Angles	R is the midpoint of QT
$\overline{QU} \parallel \overline{TS}$	Alternate Interior Angles	Given

Cut N' Paste = Proof #5

Diags of a //ogram bisect each other	$\angle TWU \cong \angle VWS$	Given
Diags of a //ogram bisect each other	$\overline{WU} \cong \overline{WS}$	$\square UTW \cong \square SVW$
$\square STUV$ is a parallelogram	Vertical Angles	$\overline{WT} \cong \overline{WV}$
SAS	$\square STUV$ is a parallelogram	Given

Other possible solutions to proof #5 (extension, day 1)



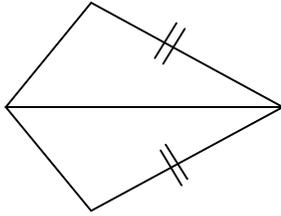
Missing Parts

Name: _____

Date: _____

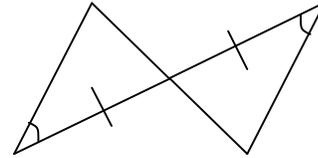
Directions: For each picture, decide what other parts need to be congruent in order to prove the triangles congruent using the given postulate or theorem. Mark the correct congruent parts on your picture.

1.



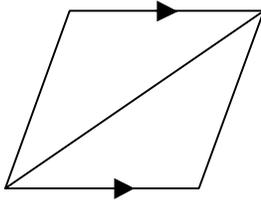
Using SSS

2.



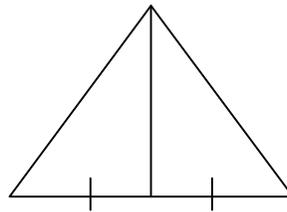
Using SAS

3.



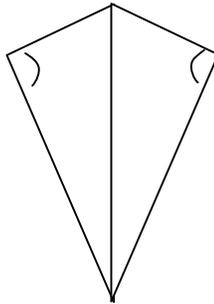
Using ASA

4.



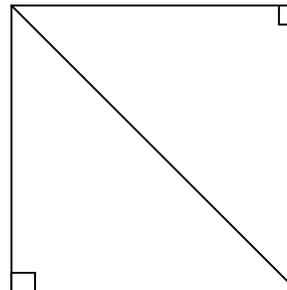
Using HL

5.



Using AAS

6.



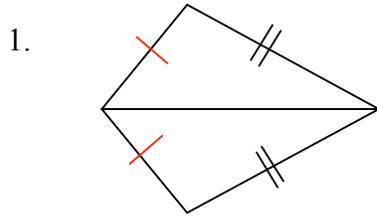
Using HL

Missing Parts

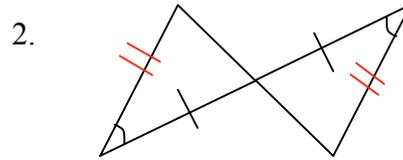
Name: ANSWER KEY

Date: _____

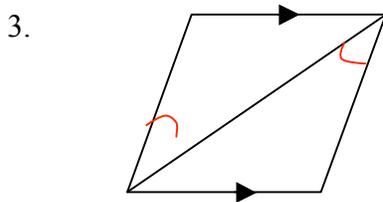
Directions: For each picture, decide what other parts need to be congruent in order to prove the triangles congruent using the given postulate or theorem. Mark the correct congruent parts on your picture.



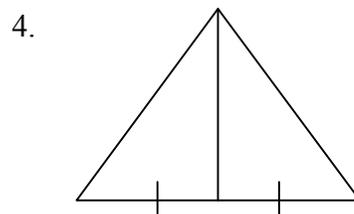
Using SSS



Using SAS

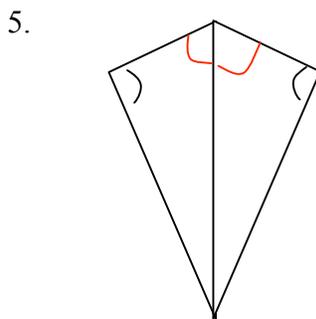


Using ASA



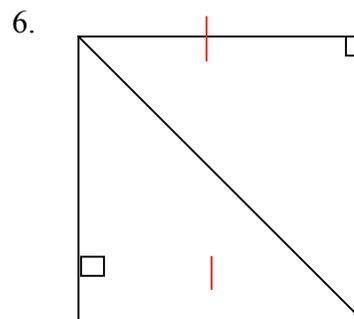
Using HL

The triangles need to be right triangles



Using AAS

The third angle is another alternative



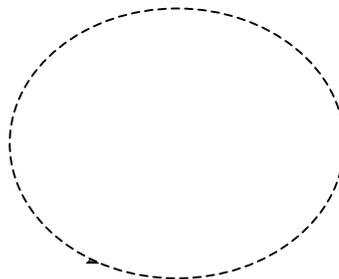
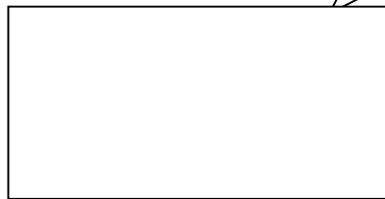
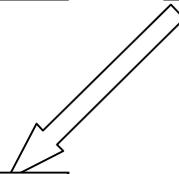
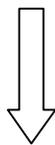
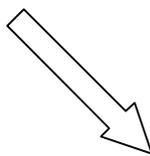
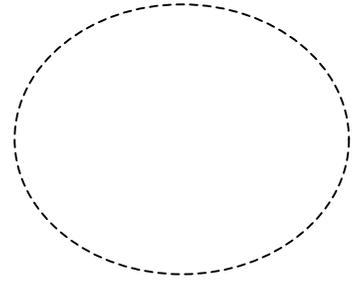
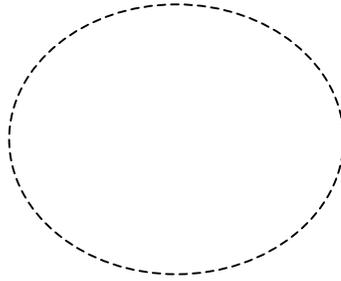
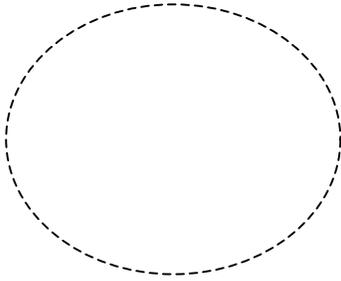
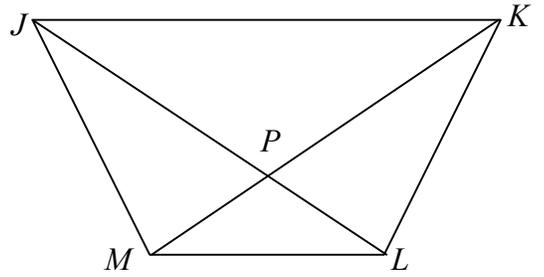
Using HL

The left and right sides could also be congruent.

Proof #6

Given: $\triangle PML$ is isosceles
 $\angle JLK$ and $\angle KMJ$ are right angles

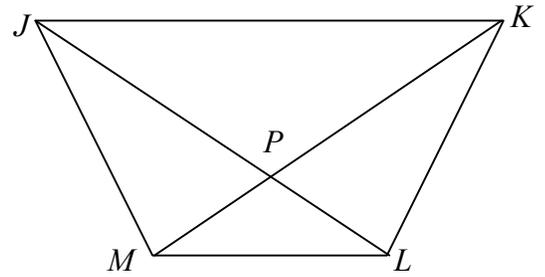
Prove: $\triangle JMP \cong \triangle KLP$



Proof #6 ANSWER KEY

Given: $\triangle PML$ is isosceles
 $\angle JLK$ and $\angle KMJ$ are right angles

Prove: $\triangle JMP \cong \triangle KLP$

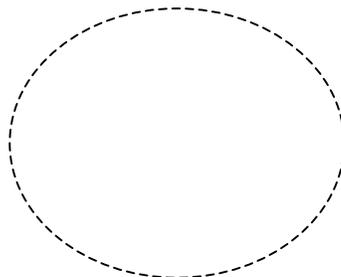
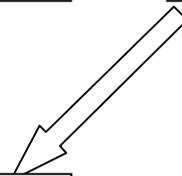
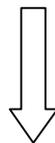
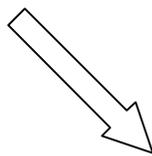
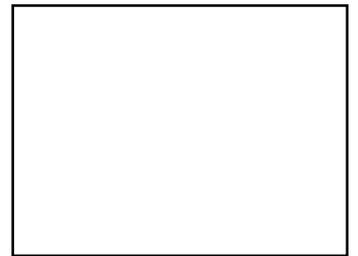
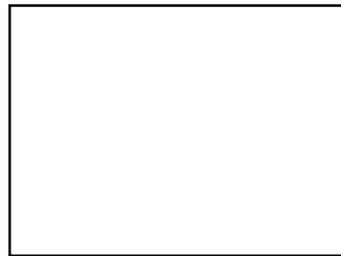
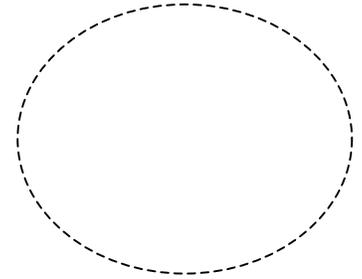
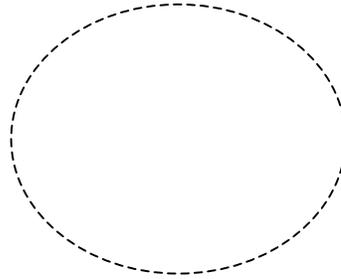
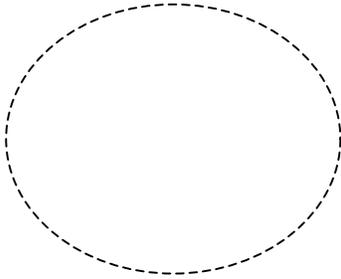
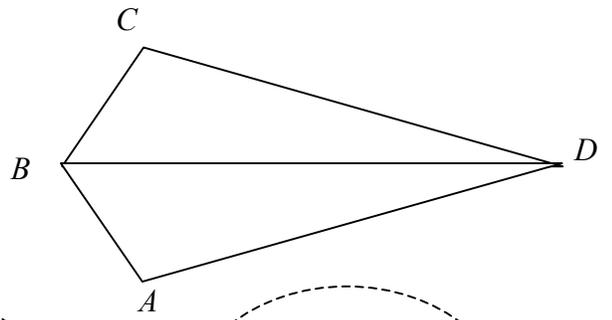


<div style="border: 1px dashed black; border-radius: 50%; width: 150px; height: 150px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\triangle PML$ is isosceles </div> <p style="text-align: center;">Given</p> <hr style="width: 100%;"/> <div style="text-align: center;"> </div> <div style="border: 1px solid black; width: 150px; height: 100px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\overline{PM} \cong \overline{PL}$ </div> <p style="text-align: center;">Defn. Isosceles Triangle</p>	<div style="border: 1px dashed black; border-radius: 50%; width: 150px; height: 150px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\angle JLK$ and $\angle KMJ$ are right angles </div> <p style="text-align: center;">Given</p> <hr style="width: 100%;"/> <div style="text-align: center;"> </div> <div style="border: 1px solid black; width: 150px; height: 100px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\angle JLK \cong \angle KMJ$ </div> <p style="text-align: center;">All right angles are congruent</p>	<div style="border: 1px dashed black; border-radius: 50%; width: 150px; height: 150px; margin: 0 auto;"></div> <hr style="width: 100%;"/> <div style="text-align: center;"> </div> <div style="border: 1px solid black; width: 150px; height: 100px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\angle JPM \cong \angle KPL$ </div> <p style="text-align: center;">Vertical Angles</p>
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div>		
<div style="border: 1px solid black; width: 150px; height: 100px; margin: 0 auto; display: flex; align-items: center; justify-content: center;"> $\triangle JMP \cong \triangle KLP$ </div> <p style="text-align: center;">AAS</p> <hr style="width: 100%;"/> <div style="text-align: center;"> </div> <div style="border: 1px dashed black; border-radius: 50%; width: 150px; height: 150px; margin: 0 auto;"></div>		

Proof #7

Given: $\angle CBD \cong \angle ABD$
 $\angle C \cong \angle A$

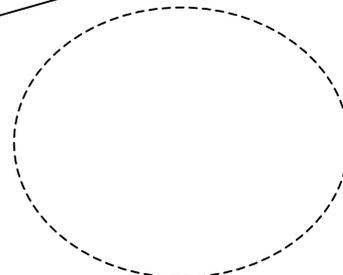
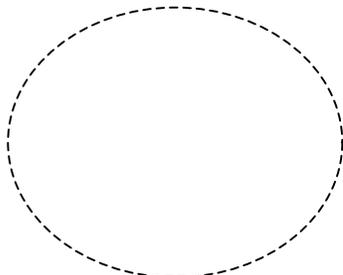
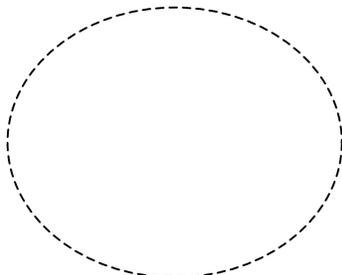
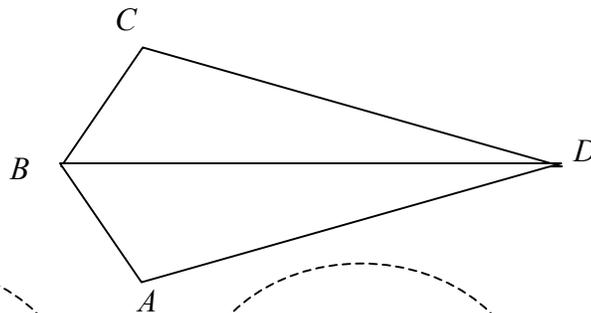
Prove: $\angle CDB \cong \angle ADB$ using triangle congruence



Proof #7 ANSWER KEY

Given: $\angle CBD \cong \angle ABD$
 $\angle C \cong \angle A$

Prove: $\angle CDB \cong \angle ADB$ using triangle congruence



$\angle CBD \cong \angle ABD$

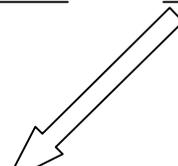
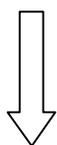
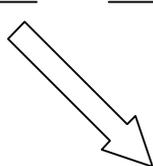
$\angle C \cong \angle A$

$\overline{BD} \cong \overline{BD}$

Given

Given

Reflexive Property



$\triangle CBD \cong \triangle ABD$

AAS

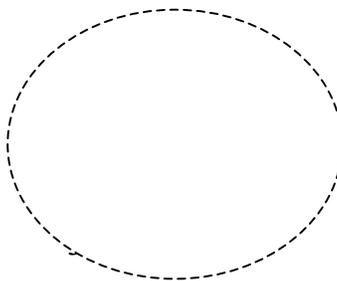
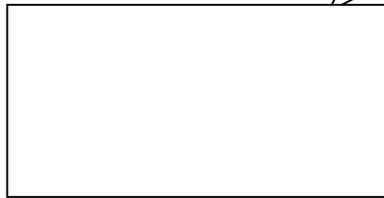
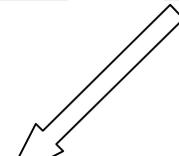
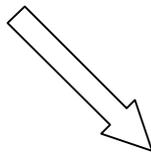
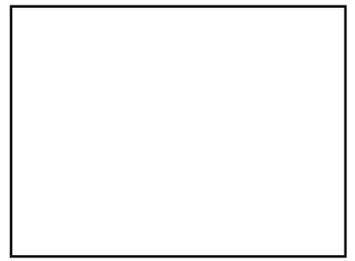
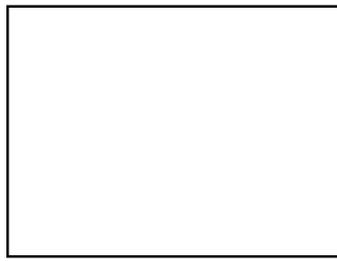
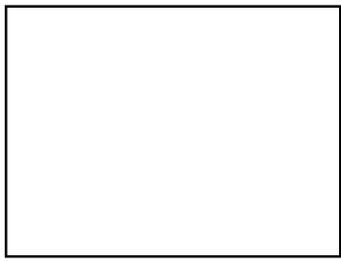
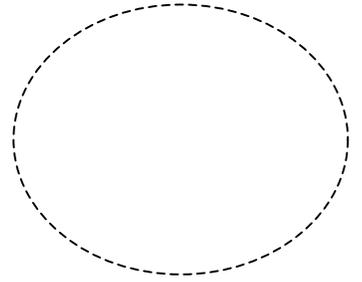
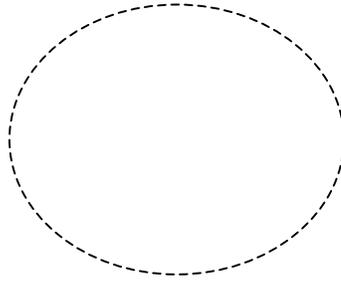
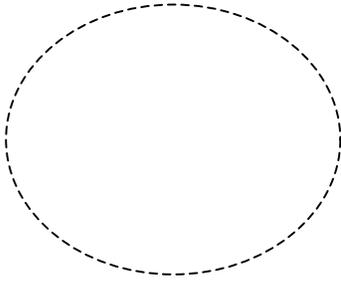
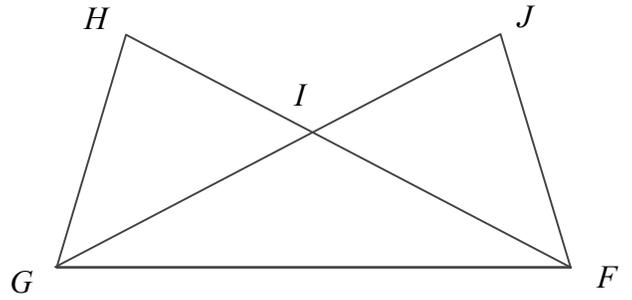
$\angle CDB \cong \angle ADB$

CPCTC

Proof #8

Given: $\overline{HG} \cong \overline{JF}$
 $\overline{HF} \cong \overline{JG}$

Prove: $\angle JGF \cong \angle HFG$

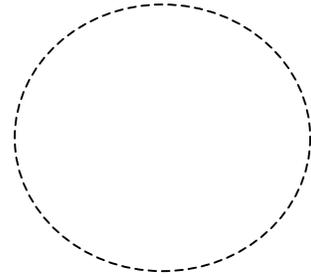
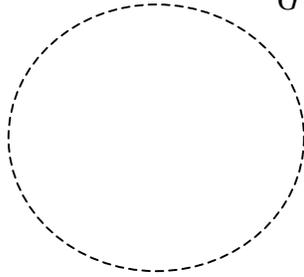
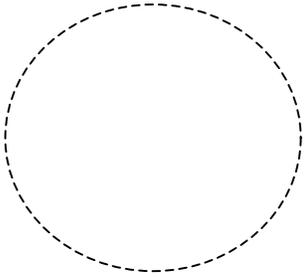
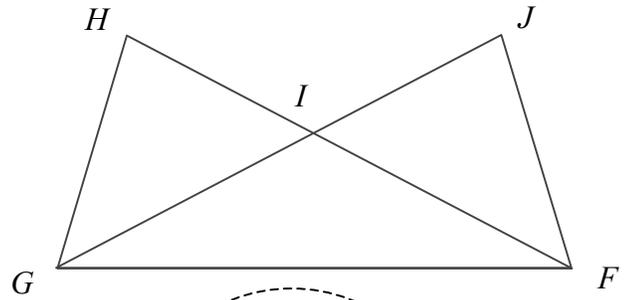


Proof #8 ANSWER KEY

Given: $\overline{HG} \cong \overline{JF}$

$\overline{HF} \cong \overline{JG}$

Prove: $\angle JGF \cong \angle HFG$



$\overline{HG} \cong \overline{JF}$

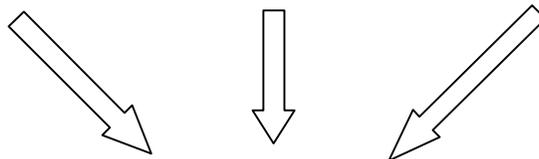
$\overline{HF} \cong \overline{JG}$

$\overline{GF} \cong \overline{GF}$

Given

Given

Reflexive Property



$\triangle HGF \cong \triangle JFG$

SSS



$\angle JGF \cong \angle HFG$

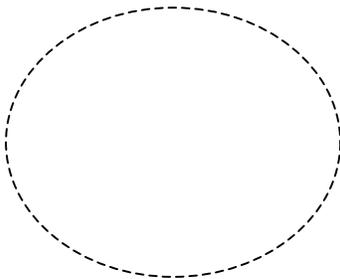
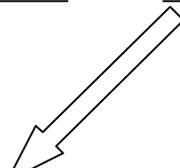
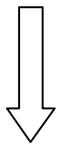
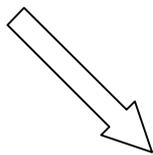
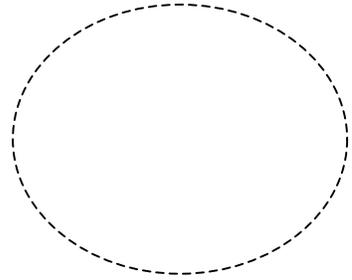
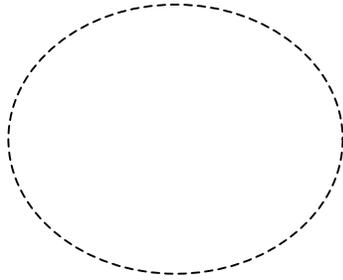
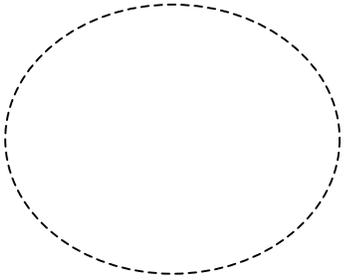
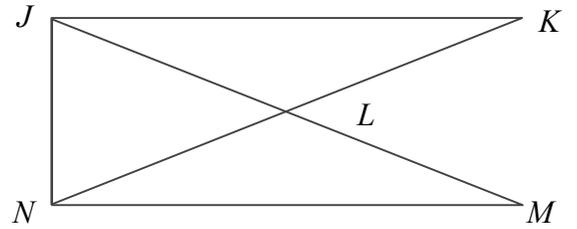
CPCTC

Proof #9

Given: $\angle KJN$ and $\angle MNJ$ are right angles

$$\overline{JM} \cong \overline{NK}$$

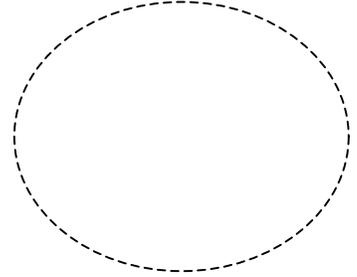
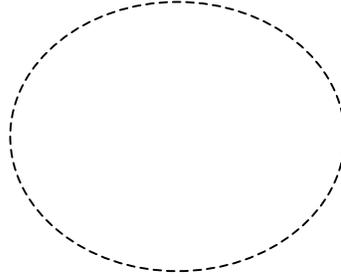
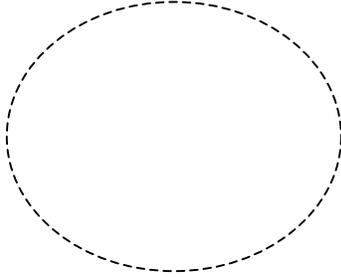
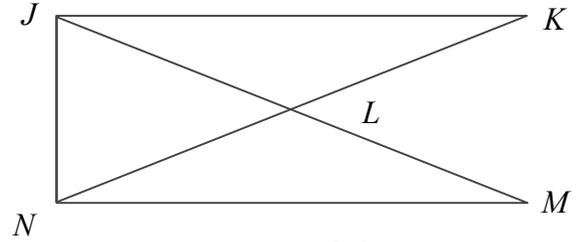
Prove: $\angle MJN \cong \angle KNJ$



Proof #9 ANSWER KEY

Given: $\angle KJN$ and $\angle MNJ$ are right angles
 $\overline{JM} \cong \overline{NK}$

Prove: $\angle MJN \cong \angle KNJ$



$\angle KJN$ and $\angle MNJ$
are right angles

Given



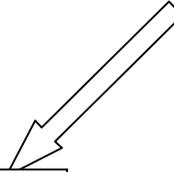
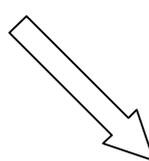
$\overline{JM} \cong \overline{NK}$

Given



$\overline{JN} \cong \overline{JN}$

Reflexive



$\triangle JKN \cong \triangle MNK$

HL



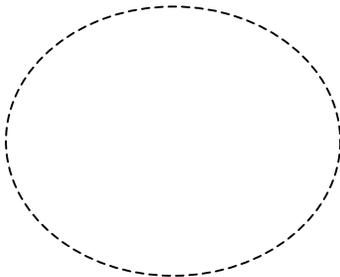
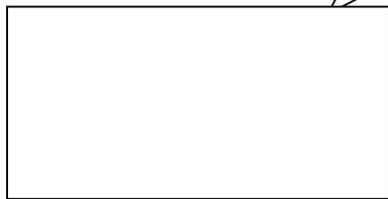
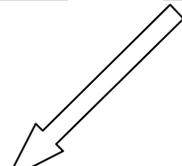
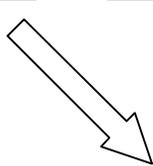
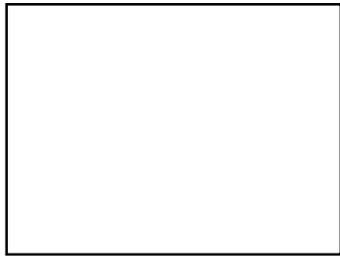
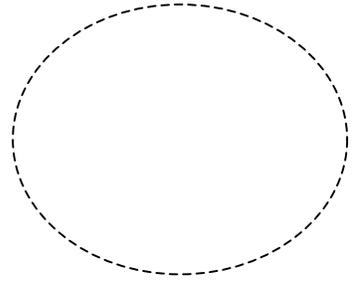
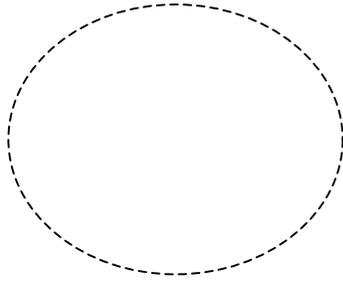
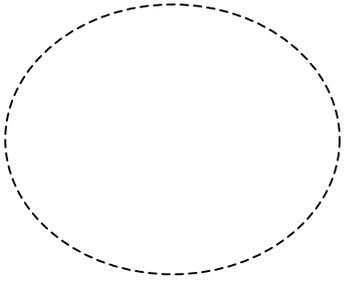
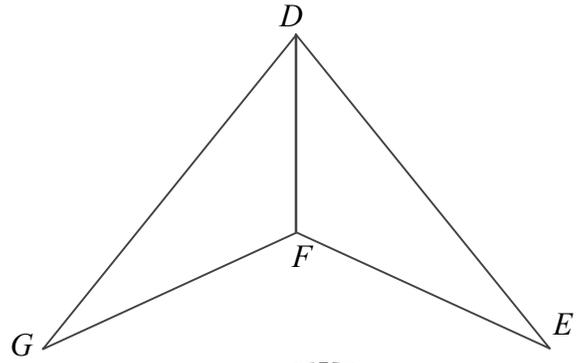
$\angle MJN \cong \angle KNJ$

CPTC

Proof #10

Given: \overline{DF} bisects $\angle GDE$
 $\overline{DG} \cong \overline{DE}$

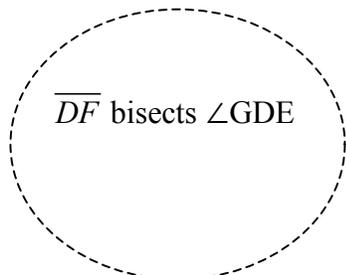
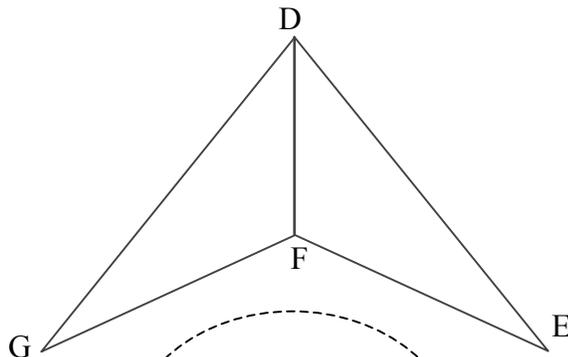
Prove: $\angle G \cong \angle E$



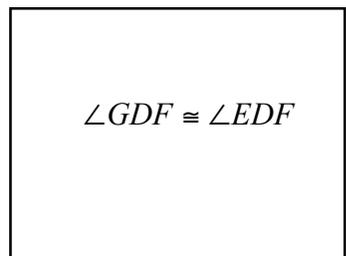
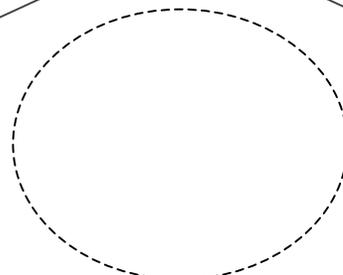
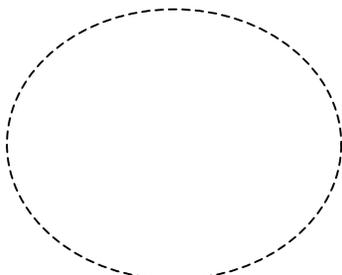
Proof #10 ANSWER KEY

Given: \overline{DF} bisects $\angle GDE$
 $\overline{DG} \cong \overline{DE}$

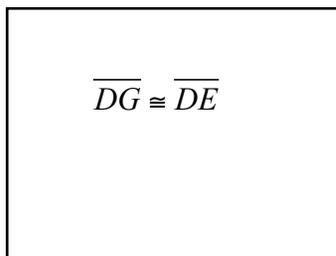
Prove: $\angle G \cong \angle E$



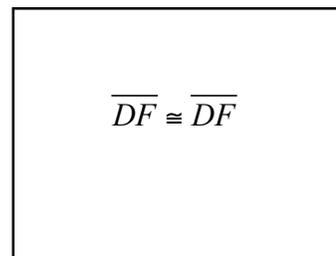
Given



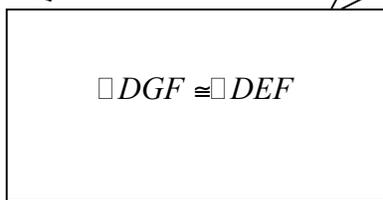
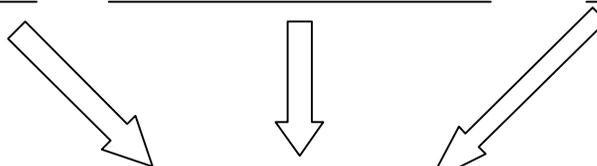
Defn. of Angle Bisector



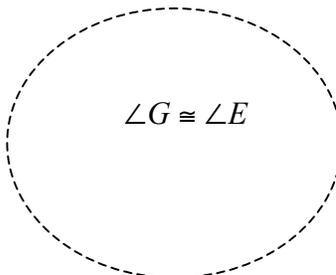
Given



Reflexive Property



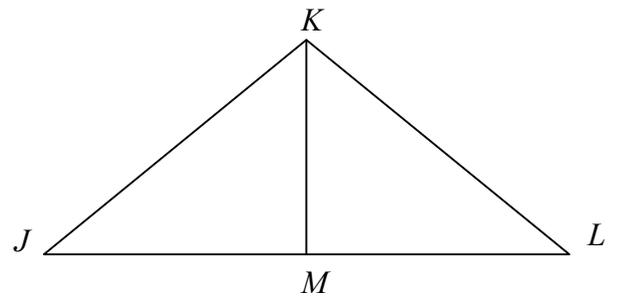
SAS



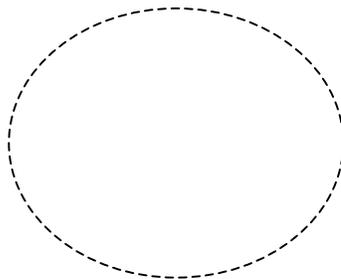
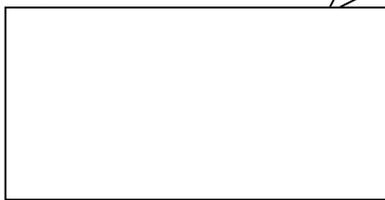
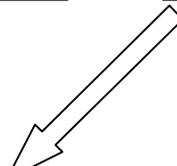
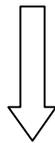
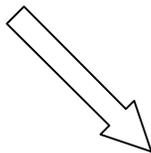
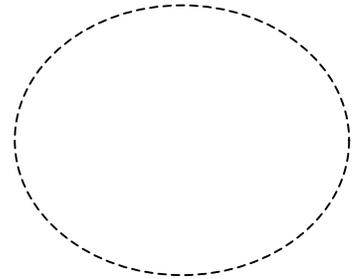
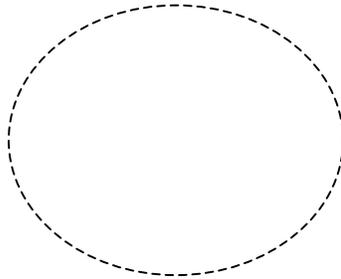
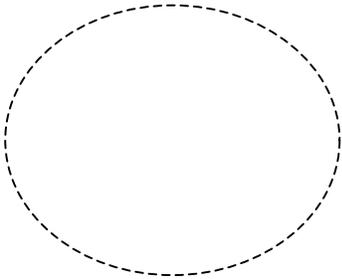
CPCTC

Proof #11

Given: $\triangle JKL$ is an isosceles triangle
 \overline{KM} is an angle bisector



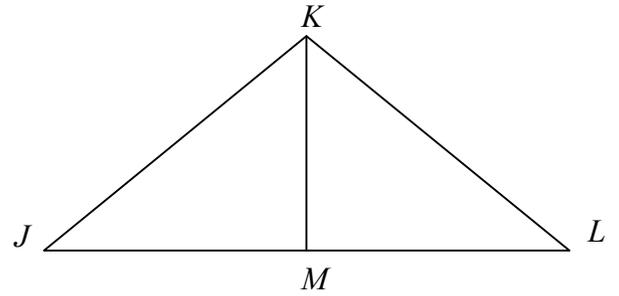
Prove: $\overline{JM} \cong \overline{LM}$



Proof #11 ANSWER KEY

Given: $\triangle JKL$ is an isosceles triangle
 \overline{KM} is an angle bisector

Prove: $\overline{JM} \cong \overline{LM}$



$\triangle JKL$ is an isosceles triangle

Given

\overline{KM} is an angle bisector

Given



A: $\angle J \cong \angle L$
 Or
 B: $\overline{JK} \cong \overline{LK}$

A: Isos. Triangle Theorem

Or

B: Defn. Isos. Triangle



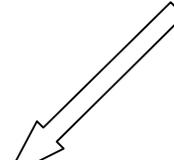
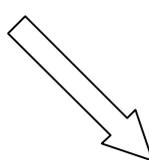
$\angle JKM \cong \angle LKM$

Defn. Angle Bisector



$\overline{KM} \cong \overline{KM}$

Reflexive Property



$\triangle JKM \cong \triangle LKM$

A: AAS or B: SAS

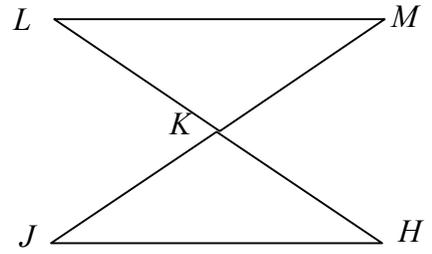


$\overline{JM} \cong \overline{LM}$

CPCTC

Proof #12

Given: \overline{LH} and \overline{JM} bisect each other at K



Prove: $\overline{HJ} \cong \overline{ML}$

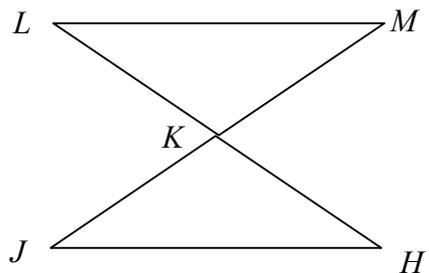
A series of boxes and arrows for a proof. It starts with three dashed circles, followed by three solid rectangles, then a central solid rectangle, and finally a dashed circle at the bottom. Arrows indicate the flow of the proof steps.

The diagram shows a sequence of boxes and arrows for a proof. It starts with three dashed circles, followed by three solid rectangles, then a central solid rectangle, and finally a dashed circle at the bottom. Arrows indicate the flow of the proof steps.

Proof #12 ANSWER KEY

Given: \overline{LH} and \overline{JM} bisect each other at K

Prove: $\overline{HJ} \cong \overline{ML}$



\overline{LH} and \overline{JM} bisect each other at K

\overline{LH} and \overline{JM} bisect each other at K



$\overline{LK} \cong \overline{HK}$

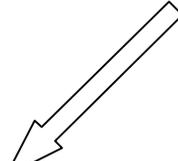
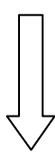
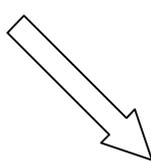
$\overline{MK} \cong \overline{JK}$

$\angle LKM \cong \angle HKJ$

Defn. of a bisector

Defn. of a bisector

Vertical Angles



$\triangle LKM \cong \triangle HKJ$

SAS



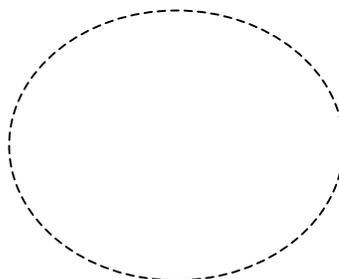
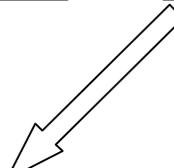
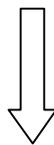
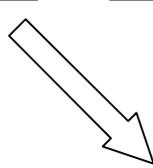
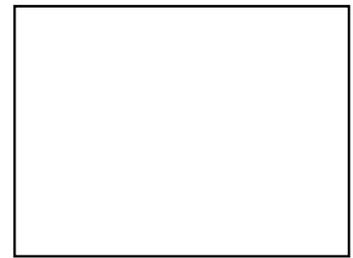
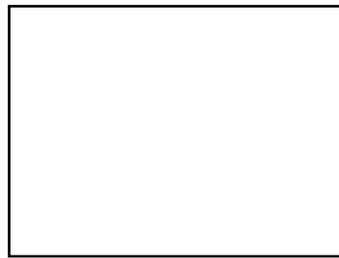
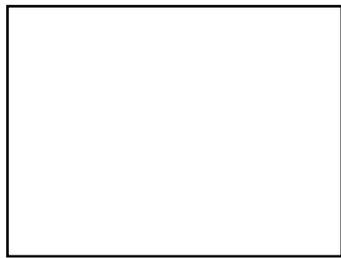
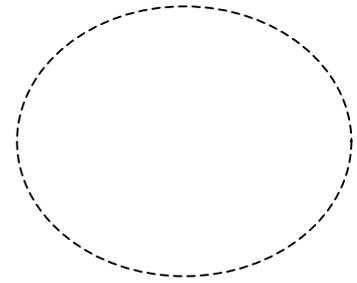
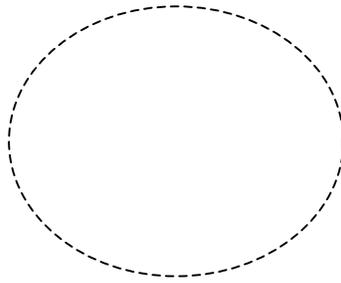
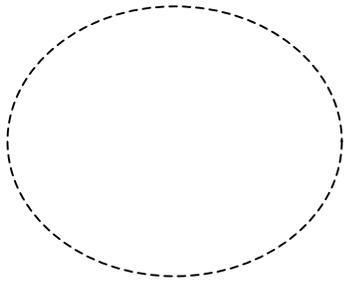
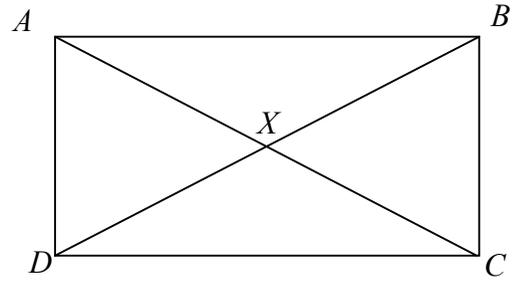
$\overline{HJ} \cong \overline{ML}$

CPCTC

Proof #13

Given: $ABCD$ is a rectangle

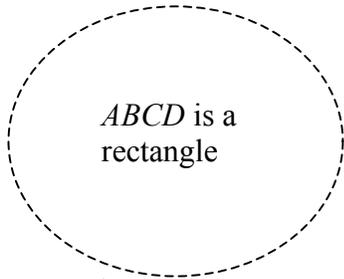
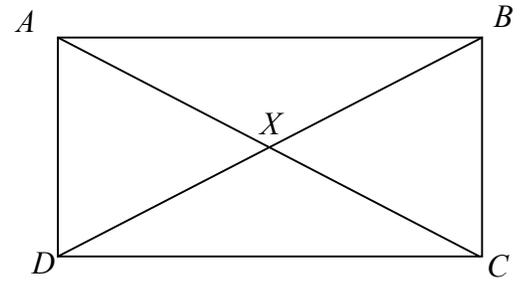
Prove: $\angle BDC \cong \angle ACD$



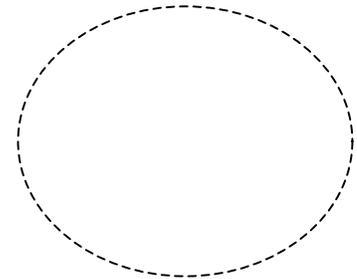
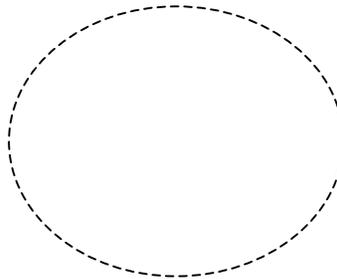
Proof #13 ANSWER KEY

Given: $ABCD$ is a rectangle

Prove: $\angle BDC \cong \angle ACD$



Given



$\angle ADC$ and $\angle BCD$ are right angles

Defn. of a rectangle



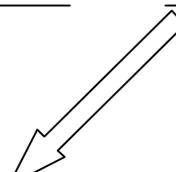
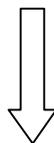
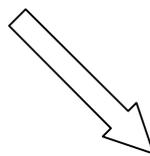
$\overline{AC} \cong \overline{BD}$

Diags. of a rectangle are congruent



$\overline{CD} \cong \overline{CD}$
or
 $\overline{AD} \cong \overline{BC}$

Reflexive or
Opp. sides of a rectangle are congruent



$\triangle ADC \cong \triangle BCD$

HL



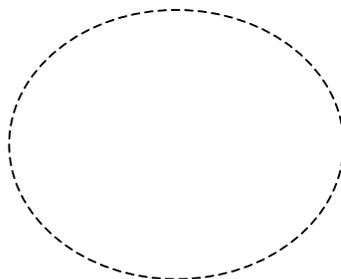
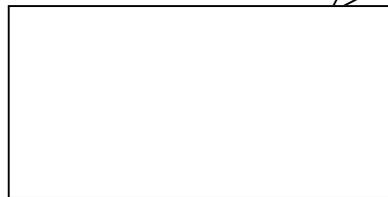
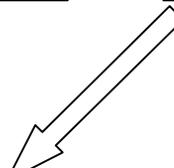
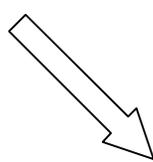
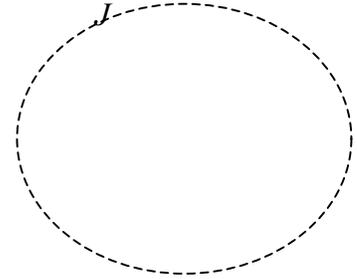
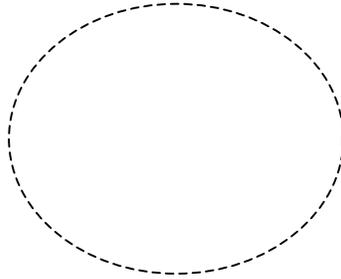
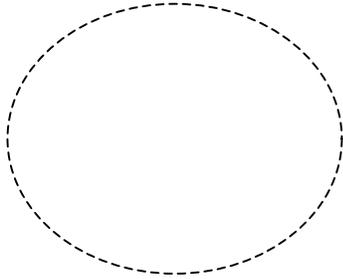
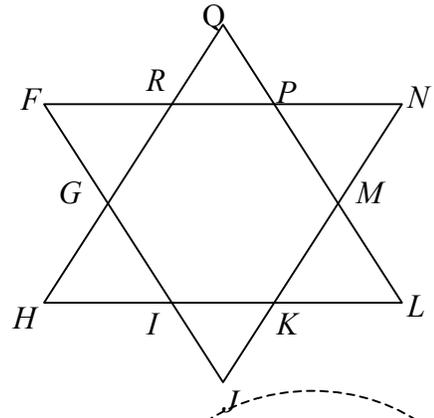
$\angle BDC \cong \angle ACD$

CPCTC

Proof #14

Given: $RPMKIG$ is a regular hexagon
 $\square FNJ$ is equilateral

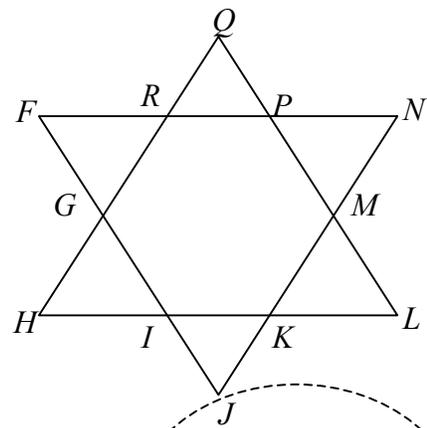
Prove: $\angle FGR \cong \angle NMP$



Proof #14 ANSWER KEY

Given: $RPMKIG$ is a regular hexagon
 $\square FNJ$ is equilateral

Prove: $\angle FGR \cong \angle NMP$



RPMKIG is a regular hexagon

Given



$\overline{GR} \cong \overline{PM}$

Defn. regular polygons

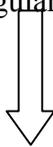
$\square FNJ$ is equilateral

Given



$\angle F \cong \angle N$

Equilateral triangles are equiangular



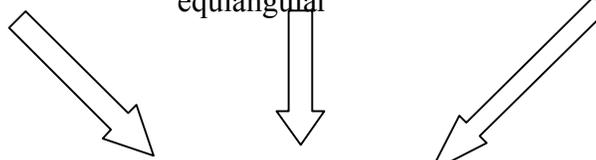
$\angle GRP \cong \angle MPR$

Defn. regular polygons



$\angle FRG \cong \angle NPM$

Supplements of congruent angles are congruent



$\square FRG \cong \square NPM$

AAS



$\angle FGR \cong \angle NMP$

CPCTC

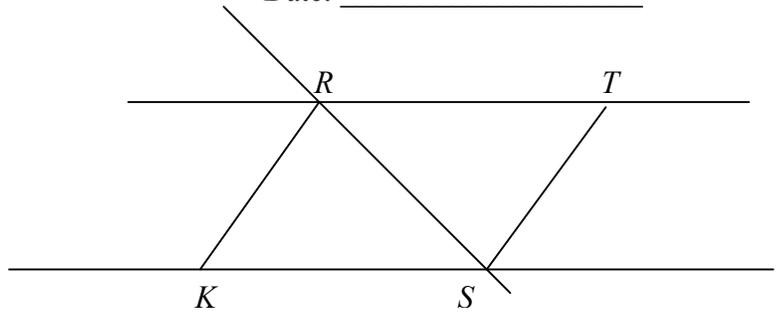
Two Column Proofs

Name: _____
Date: _____

1.

Given: $\overline{RT} \parallel \overline{KS}$, $\overline{KR} \parallel \overline{ST}$

Prove: $\triangle KRS \cong \triangle TSR$

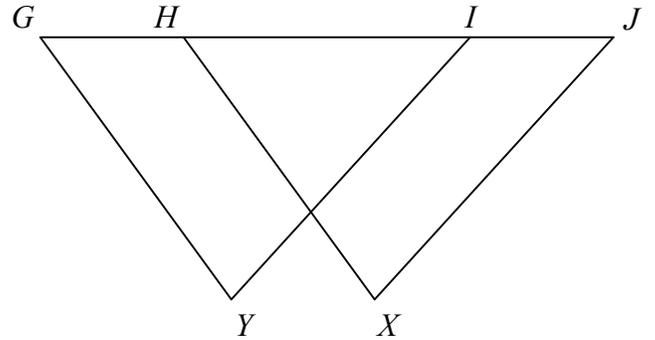


STATEMENTS	REASONS
1. $\overline{RT} \parallel \overline{KS}$	1.
2. $\angle TRS \cong \angle RSK$	2.
3. $\overline{KR} \parallel \overline{ST}$	3.
4. $KRTS$ is a parallelogram	4.
5. $\angle RKS \cong \angle STR$	5.
6. $\overline{RS} \cong \overline{RS}$	6.
7. $\triangle KRS \cong \triangle TSR$	7.

2.

Given: $\overline{GH} \cong \overline{IJ}$, $\overline{GY} \parallel \overline{HX}$, $\overline{IY} \parallel \overline{JX}$

Prove: $\triangle GYI \cong \triangle HXJ$



STATEMENTS	REASONS
1.	1. Given
2.	2. Additive Property
3.	3. Given
4.	4. Definition of Corresponding Angles
5.	5. Given
6.	6. Definition of Corresponding Angles
7.	7. ASA

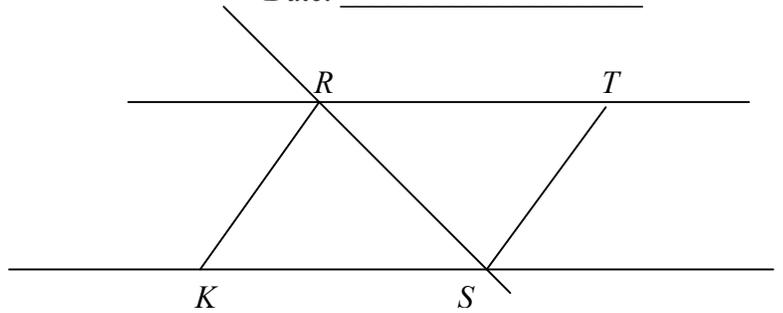
Two Column Proofs

Name: ANSWER KEY
Date:

1.

Given: $\overline{RT} \parallel \overline{KS}$, $\overline{KR} \parallel \overline{ST}$

Prove: $\triangle KRS \cong \triangle TSR$

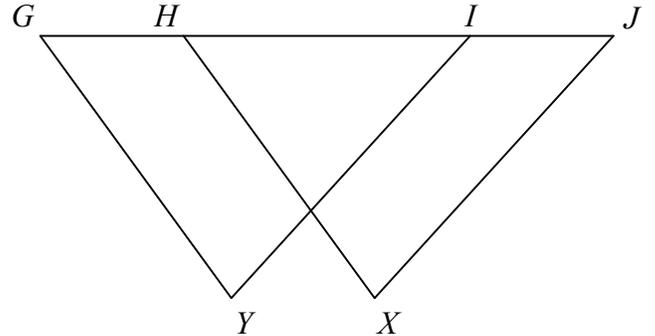


STATEMENTS	REASONS
1. $\overline{RT} \parallel \overline{KS}$	1. Given
2. $\angle TRS \cong \angle RSK$	2. Definition of Alt. Interior Angles
3. $\overline{KR} \parallel \overline{ST}$	3. Given
4. $KRTS$ is a parallelogram	4. Def. of Parallelogram
5. $\angle RKS \cong \angle STR$	5. Property of Parallelogram
6. $\overline{RS} \cong \overline{RS}$	6. Reflexive Property
7. $\triangle KRS \cong \triangle TSR$	7. AAS

2.

Given: $\overline{GH} \cong \overline{IJ}$, $\overline{GY} \parallel \overline{HX}$, $\overline{IY} \parallel \overline{JX}$

Prove: $\triangle GYI \cong \triangle HXJ$

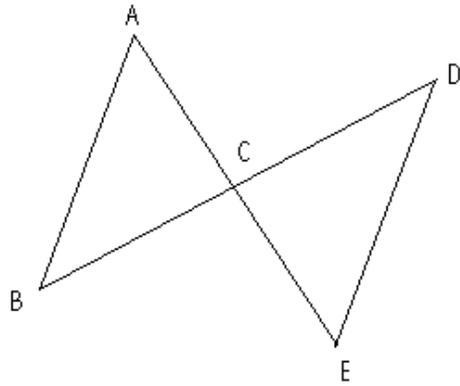


STATEMENTS	REASONS
1. $\overline{GH} \cong \overline{IJ}$	1. Given
2. $\overline{GI} \cong \overline{HJ}$	2. Additive Property
3. $\overline{GY} \parallel \overline{HX}$	3. Given
4. $\angle IGY \cong \angle JHX$	4. Definition of Corresponding Angles
5. $\overline{IY} \parallel \overline{JX}$	5. Given
6. $\angle GIY \cong \angle HJX$	6. Definition of Corresponding Angles
7. $\triangle GYI \cong \triangle HXJ$	7. ASA

Proof # 15

Given: $\overline{AB} \parallel \overline{ED}$, and C is the midpoint of \overline{BD}

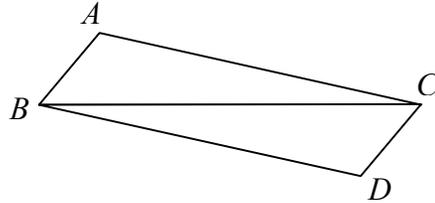
Prove: $\triangle BCA \cong \triangle DCE$



Proof # 16

Given: $\overline{AB} \cong \overline{DC}$, $\overline{AC} \cong \overline{DB}$

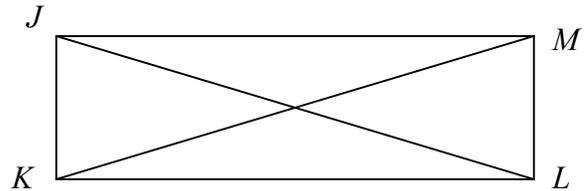
Prove: $\triangle CBD \cong \triangle BCA$



Proof # 17

Given: $\square JKLM$ is a rectangle

Prove: $\overline{JL} \cong \overline{MK}$



Proof # 18

Given: $\square FGHI$ is a kite, with $\overline{FG} \cong \overline{GH}$

Prove: $\triangle FIG \cong \triangle HIG$

Proof # 19

Given: \overline{LM} and \overline{NO} bisect each other at P

Prove: $\overline{LN} \cong \overline{OM}$

Proof # 15 ANSWER KEYGiven: $\overline{AB} \parallel \overline{ED}$, and C is the midpoint of \overline{BD} Prove: $\triangle BCA \cong \triangle DCE$

STATEMENTS	REASONS
1. $\overline{AB} \parallel \overline{ED}$	1. Given
2. $\angle ABC \cong \angle EDC$	2. Def. of Alt. Int. Angles
3. C is the midpoint of \overline{BD}	3. Given
4. $\overline{BC} \cong \overline{DC}$	4. Def. of Midpoint
5. $\angle ACB \cong \angle ECD$	5. Vertical Angles
6. $\triangle BCA \cong \triangle DCE$	6. ASA

Proof # 16 ANSWER KEYGiven: $\overline{AB} \cong \overline{DC}$, $\overline{AC} \cong \overline{DB}$ Prove: $\triangle CBD \cong \triangle BCA$

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DC}$	1. Given
2. $\overline{AC} \cong \overline{DB}$	2. Given
3. $\overline{BC} \cong \overline{BC}$	3. Reflexive Property
4. $\triangle ABC \cong \triangle DCB$	4. SSS
5. $\triangle CBD \cong \triangle BCA$	5. CPCTC

Proof # 17 ANSWER KEYGiven: $\square JKLM$ is a rectangleProve: $\overline{JL} \cong \overline{MK}$ using congruent triangles

STATEMENTS	REASONS
1. $\square JKLM$ is a rectangle	1. Given
2. $\overline{JK} \cong \overline{ML}$	2. Definition of a rectangle
3. $\angle JKL \cong \angle MLK$	3. Definition of a rectangle
4. $\overline{KL} \cong \overline{KL}$	4. Reflexive
5. $\triangle JKL \cong \triangle MLK$	5. SAS
6. $\overline{JL} \cong \overline{MK}$	6. CPCTC

Proof # 18 ANSWER KEYGiven: $\square FGHI$ is a kite, with $\overline{FG} \cong \overline{HG}$ Prove: $\square FIG \cong \square HIG$

STATEMENTS	REASONS
1. $\square FGHI$ is a kite	1. Given
2. $\overline{FG} \cong \overline{HG}$	2. Given
3. $\overline{FI} \cong \overline{HI}$	3. Definition of a Kite
4. $\overline{GI} \cong \overline{GI}$	4. Reflexive
5. $\square FGI \cong \square HGI$	5. SSS
6. $\square FIG \cong \square HIG$	6. CPCTC

Proof # 19 ANSWER KEYGiven: \overline{LM} and \overline{NO} bisect each other at P Prove: $\overline{LN} \cong \overline{OM}$

STATEMENTS	REASONS
1. \overline{LM} and \overline{NO} bisect each other at P	1. Given
2. $\overline{LP} \cong \overline{PM}$, $\overline{NP} \cong \overline{PO}$	2. Definition of Bisect
3. $\square LPN \cong \square OPM$	3. Vertical Angles
4. $\square LPN \cong \square MPO$	4. SAS
5. $\overline{LN} \cong \overline{OM}$	5. CPCTC

Round Table Proofs

Student #1: _____
 Student #3: _____

Student #2 _____
 Student #4 _____

Given: $\overline{AB} \cong \overline{BC}, \overline{BD} \perp \overline{AB}$

Picture:

Prove: $\triangle ABD \cong \triangle CBD$

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{BC}$	1.
2.	2. Isosceles Triangle Theorem
3.	3. Given
4. $\angle BDA \cong \angle BDC$	4.
5.	5. Reflexive property
6. $\triangle ABD \cong \triangle CBD$	6.

Student #1: _____
 Student #3: _____

Student #2 _____
 Student #4 _____

Given: $\overline{EF} \parallel \overline{HI}, G$ is the midpoint of \overline{EI}

Picture:

Prove: $\triangle EGF \cong \triangle IGH$

STATEMENTS	REASONS
1. G is the midpoint of \overline{EI}	1.
2.	2. Definition of midpoint
3.	3. Given
4. $\angle FEG \cong \angle HIG$	4.
5. $\angle EGF \cong \angle IGH$	5.
6. $\triangle EGF \cong \triangle IGH$	6.

Student #1: _____

Student #2 _____

Student #3: _____

Student #4 _____

Given: $ABCD$ is a parallelogram
with diagonal \overline{DB}

Picture:

Prove: $\angle ADB \cong \angle CBD$

STATEMENTS	REASONS
1. $ABCD$ is a parallelogram w/ diag \overline{DB}	1.
2. $\overline{AD} \cong \overline{CB}, \overline{AB} \cong \overline{CD}$	2.
3.	3.
4.	4. SSS
5. $\angle ADB \cong \angle CBD$	5.

Student #1: _____

Student #2 _____

Student #3: _____

Student #4 _____

Given: \overline{PR} and \overline{QS} bisect each other at T

Picture:

Prove: $\overline{PQ} \cong \overline{SR}$

STATEMENTS	REASONS
1.	1. Given
2. $\overline{PT} \cong \overline{TR}, \overline{QT} \cong \overline{TS}$	2.
3.	3.
4.	4. SAS
5. $\overline{PQ} \cong \overline{SR}$	5.

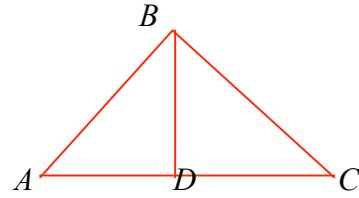
Round Table Proofs ANSWER KEY

Student #1: _____
 Student #3: _____

Student #2 _____
 Student #4 _____

Given: $\overline{AB} \cong \overline{BC}, \overline{BD} \perp \overline{AC}$

Picture:



Prove: $\triangle ABD \cong \triangle CBD$

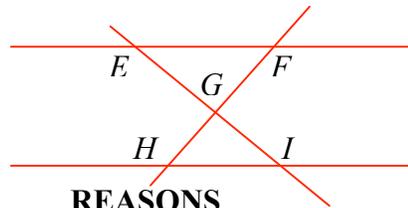
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{BC}$	1. Given
2. $\angle BAD \cong \angle BCD$	2. Isosceles Triangle Theorem
3. $\overline{BD} \perp \overline{AC}$	3. Given
4. $\angle BDA \cong \angle BDC$	4. Definition of Perpendicular
5. $\overline{BD} \cong \overline{BD}$	5. Reflexive property
6. $\triangle ABD \cong \triangle CBD$	6. AAS

Student #1: _____
 Student #3: _____

Student #2 _____
 Student #4 _____

Given: $\overline{EF} \parallel \overline{HI}, G$ is the midpoint of \overline{EI}

Picture:



Prove: $\triangle EGF \cong \triangle IGH$

STATEMENTS	REASONS
1. G is the midpoint of \overline{EI}	1. Given
2. $\overline{EG} \cong \overline{GI}$	2. Definition of midpoint
3. $\overline{EF} \parallel \overline{HI}$	3. Given
4. $\angle FEG \cong \angle HIG$	4. Alt. Int. Angles
5. $\angle EGF \cong \angle IGH$	5. Vertical Angles
6. $\triangle EGF \cong \triangle IGH$	6. ASA

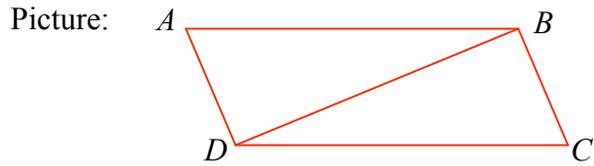
Student #1: _____

Student #2 _____

Student #3: _____

Student #4 _____

Given: $ABCD$ is a parallelogram
with diagonal \overline{DB}



Prove: $\angle ADB \cong \angle CBD$

STATEMENTS	REASONS
1. $ABCD$ is a parallelogram w/ diag \overline{DB}	1. Given
2. $\overline{AD} \cong \overline{CB}, \overline{AB} \cong \overline{CD}$	2. Property of Parallelogram
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive Property
4. $\triangle ADB \cong \triangle CBD$	4. SSS
5. $\angle ADB \cong \angle CBD$	5. CPCTC

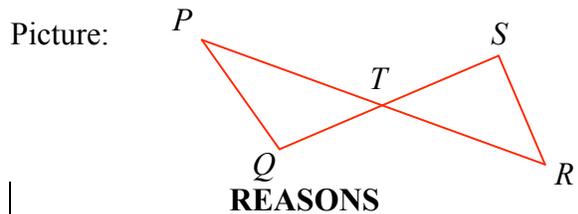
Student #1: _____

Student #2 _____

Student #3: _____

Student #4 _____

Given: \overline{PR} and \overline{QS} bisect each other at T



Prove: $\overline{PQ} \cong \overline{SR}$

STATEMENTS	REASONS
1. \overline{PR} and \overline{QS} bisect each other at T	1. Given
2. $\overline{PT} \cong \overline{TR}, \overline{QT} \cong \overline{TS}$	2. Definition of Bisect
3. $\angle PTQ \cong \angle RTS$	3. Vertical Angles
4. $\triangle PTQ \cong \triangle RTS$	4. SAS
5. $\overline{PQ} \cong \overline{SR}$	5. CPCTC

Flow Chart Template

